Mirror-symmetry in images and 3D shapes

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Symmetry is all around...

**In nature:** mirror, rotational, helical, scale (fractal)

![Image of butterfly, flower, DNA, and landscape]

**Man-made objects:** mirror, rotational, quadrilateral

![Image of Taj Mahal, pottery, and carpet]

**Music:** translation, glide reflection

Beethoven’s *Moonlight* Sonata

Chopin’s *Waltz*
Symmetry in Image and 3D Shape Analysis

- Object recognition in human perception is greatly enhanced by the presence of symmetries [BR79]

- Automatic object recognition in images and 3D shape matching work better in the presence of symmetries?
Object recognition in human perception is greatly enhanced by the presence of symmetries [BR79]

Automatic *object recognition in images* and *3D shape matching* work better in the presence of symmetries?

**Outline:**

II  Symmetry in images

III  Symmetry in 3D shapes

IV  Putting all together
PART II: Symmetry in images

Object recognition in images

- Recognition rate improves when symmetry information is available [PLC+12]

- Popular state-of-the-art methods use bag-of-words approaches based on SIFT-like descriptors

- Only few recent works try to include symmetry descriptors [HS12]
PART II: Symmetry in images

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- Reliable symmetry detection in images is difficult
  - Background clutter
  - Partial (not perfect) symmetries
  - Perspective effect, occlusions
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Object recognition in images

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Contribution: Symmetry detection in images with controlled number of false positives
Symmetry detection in images with controlled number of false positives

Joint work with R. Grompone von Gioi (ENS Cachan) and M. Ovsjanikov (École Polytechnique) – Symmetry Competition Workshop CVPR2013

Proposed method
- Mirror symmetry detection
- Orthogonal view of objects, no perspective skew
- Detection = two-stage process: candidate selection + validation
- Main concern: diminish false detections → a contrario approach
Candidate selection

Goal: No false negatives!

Candidate: image patch \((x_1, y_1, x_2, y_2, width) \sim (\rho, \theta, width, height, offset)\)

5 degrees of freedom \(\rightarrow\) exhaustive search not feasible
Candidate selection

Goal: No false negatives!

**Candidate**: image patch \((x_1, y_1, x_2, y_2, width) \sim (\rho, \theta, width, height, offset)\)

A. Reisfeld voting using SIFT features \(\rightarrow (\rho, \theta)\) [LE06]

\[
\Phi_i \quad \Phi_j
\]

\[
\Phi_i = \alpha_{ij}
\]

\[
w_{ij} = 1 - \cos(\Phi_i + \Phi_j - 2\alpha_{ij})
\]

B. Exhaustive search along \((\rho, \theta)\) axis using *integral images* \(\rightarrow (width, height, offset)\)
Validation

**Goal:** No false positives!

**Candidate:** image patch $s(x_1, y_1, x_2, y_2, \text{width}) \sim s(\rho, \theta, \text{width}, \text{height}, \text{offset})$

Is the given patch a meaningful symmetry?
Validation

Goal: No false positives!

Candidate: image patch $s(x_1, y_1, x_2, y_2, width) \sim s(\rho, \theta, width, height, offset)$

Measure for the degree of symmetry: gradient orientation error

for all the $\eta$ pairs of pixels in patch
accumulate normalised angular error

$$k(s) = \sum_{i}^{\eta} \frac{\delta_i}{\pi}$$

$k(s) \in [0, \eta]$ \begin{cases} 0, & \text{perfect symmetry} \\ \eta, & \text{worst symmetry} \end{cases}$

Need a detection threshold on $k(s)$
Validation

**Goal: No false positives!**

*A contrario theory*: formalises the *non-accidentalness principle* (“no perception in noise”) using a *multiple hypothesis testing* approach

- Random model: *null hypothesis* $\mathcal{H}_0$
- Candidate $s$: *outlier* w.r.t. $\mathcal{H}_0$? $\xrightarrow{\text{yes}} s$ – *meaningful* detection
Validation

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- Random model: null hypothesis $\mathcal{H}_0$
- Candidate $s$: *outlier* w.r.t. $\mathcal{H}_0$? $\implies s$ – meaningful detection

Random model $\mathcal{H}_0$: Gaussian white noise

**p-value**: $P_{\mathcal{H}_0}[k_X(s) \leq k(s)]$

**Number of candidates**: $N_I = (mn)^{\frac{5}{2}}$

$\varepsilon = 1 \rightarrow \text{one false positive per image}$

Meaningfulness test: $N_I \cdot \text{p-value}(s) \leq \varepsilon$
Results

- No detection in noise images
- Satisfactory results in general
Results

- No detection in noise images
- Satisfactory results in general

Human labels  | State-of-the-art  | Proposed
Results

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Human labels

State-of-the-art

Proposed
Is symmetry information **useful** and **used** in object recognition in images?

- **Conclusion**: Using symmetry information improves object recognition in images
- **Contribution**: Parameterless mirror-symmetry detection with controlled number of false positives
- **Limitations**: No perspective skew
- **Future work**: Design symmetry descriptors for object recognition tasks
PART III: Symmetry in 3D shapes

3D shape matching: Find *isometric correspondences* between shapes with *intrinsic symmetries*.

Isometries approximate well articulated motion of humans and animals

*Approximately preserve geodesic distances between pairs of points*.
PART III: Symmetry in 3D shapes

3D shape matching: Find *isometric correspondences* between shapes with *intrinsic symmetries*

Most approximately isometric shapes contain intrinsic symmetries

*Approximately preserve geodesic distances between pairs of points*
Given a pair of shapes with intrinsic symmetries, find isometric correspondences.

Approximately preserve geodesic distances between pairs of points.
PART III: Symmetry in 3D shapes

Given a pair of shapes with intrinsic symmetries, find isometric correspondences

\[
\hat{T} = \arg \min_T \sum_{x,y} |d^M(x,y) - d^N(T(x), T(y))| \quad \text{difficult}
\]
PART III: Symmetry in 3D shapes

Given a pair of shapes with intrinsic symmetries, find isometric correspondences

\[ T : M \rightarrow N \]

\[ d^M(x, y) \]

\[ d^N(T(x), T(y)) \]

At least two equally-good solutions; non-convex problem \( \leftarrow \) symmetry ambiguity
PART III: Symmetry in 3D shapes

Given a pair of shapes with intrinsic symmetries, find isometric correspondences

\[ T : M \rightarrow N \]

Symmetry flipping \( \rightarrow \) continuity issue; \textbf{symmetry makes the problem harder}
PART III: Symmetry in 3D shapes

Given a pair of shapes with intrinsic symmetries, find isometric correspondences

Existing work: coarse-to-fine approaches, based on some initial point correspondences
PART III: Symmetry in 3D shapes

Given a pair of shapes with intrinsic symmetries, find isometric correspondences

Existing work: coarse-to-fine approaches, based on some initial point correspondences → complex and error-prone [GYF11]
PART III: Symmetry in 3D shapes

Given a pair of shapes with intrinsic symmetries, find isometric correspondences

Contribution: Matching of 3D shapes with intrinsic symmetries without point correspondences
Matching of 3D shapes with intrinsic symmetries

Joint work with M. Ovsjanikov (École Polytechnique), Q. Mérigot (CNRS), L. Guibas (Stanford University) – SGP2013

**Proposed method**
- Uses *functional map* framework \([\text{OBCS}^+12]\) for shape matching
- Matching is done between halves of the shapes
- Two equally good solutions are returned
New concept of matching between shapes

- $T : N \rightarrow M$, pointwise map $T(y) = x$
- $T_F : L^2(M) \rightarrow L^2(N)$, function-wise map $T_F(f) = g$, where $g = f \circ T$
- $T \Rightarrow T_F$ and $T_F \Rightarrow T$
Functional map representation

New concept of matching between shapes

- \( T : N \rightarrow M \), pointwise map \( T(y) = x \)
- \( T_F : L^2(M) \rightarrow L^2(N) \), function-wise map \( T_F(f) = g \), where \( g = f \circ T \)
- \( T \Rightarrow T_F \) and \( T_F \Rightarrow T \)
How to compute $T_F$?

- $T_F$ – linear map between vector spaces

- Choose *convenient* bases (LB eigenfunctions) for the two vector spaces:
  
  $f = \sum_i a_i \Phi_i^M$ and $g = \sum_i b_i \Phi_i^N$

- $T_F$ – matrix representation $Ca = b$

- Given enough pairs of functions defined on the two shapes, $C$ can be recovered through a least squares system

- Function constraints: descriptor preservation (HKS, WKS), landmark correspondences

- Functional map: state-of-the-art results in isometric shape matching

- Drawback: symmetry flipping – needs some point correspondences
Quotient space matching

Given a pair of shapes with \textit{known} intrinsic symmetries, solve the symmetry ambiguity by \textit{matching between halves of the shapes}

- Quotienting the shape corresponds to splitting the function space

\[ \tilde{M} = M / S_M \]

\[ \tilde{N} = N / S_N \]
Quotient space matching

Given a pair of shapes with known symmetries:

- Split the space of functions into symmetric and antisymmetric subspaces
  \[ L^2_+(M) = \{ f \in L^2(M) \mid f \circ S_M = f \}; \quad L^2(M) = L^2_+(M) \oplus L^2_-(M) \]
- There exists an orthogonal basis of \( L^2_+(M) \) formed by LB eigenfunctions
- Functional map decomposed into parts \( C = C_+ \oplus C_- \), estimated independently

\[ L^2(M) \quad \quad L^2(N) \]

\[ C = C_+ \oplus C_- \]
Given a pair of shapes with known symmetries:

- Solve for $C_+$ as before
- $C_+$ is easier to compute than $C$
  - $C_+$ is unique
  - The descriptors (HKS, WKS) are usually symmetric functions
- Use $C_+$ to recover a point-to-orbit map
- Use the known symmetries to compute two equally-good point-to-point maps
Semi-quotient space matching

More practical scenario: the symmetry is known on only one shape

For the map $C : L^2(N) \rightarrow L^2(M)$, compute its symmetric part $R$

Transfer the symmetry, and proceed as before
Results

Symmetry transfer accuracy

State-of-the-art

Proposed method
Results

Quotient space matching

Source

BIM

Original

FMAP

Proposed method
Summary

Is symmetry information **useful** and **used** in 3D shape matching?

- **Conclusion:** Shape matching apparently more difficult for shapes with symmetries
- **Contribution:** Matching of shapes with intrinsic symmetry without point correspondences
- **Bonus:** Accurate symmetry estimation on an unknown shape through symmetry transfer
PART IV: Putting all together

- Large public repositories of images (flickr) and 3D shapes (Google Warehouse)
- Increasing interest for joint image and 3D shape analysis
- Possible application: automatic texture mapping from natural images
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