The Compressed Annotation Matrix: An Efficient Data Structure for Computing Persistent Cohomology

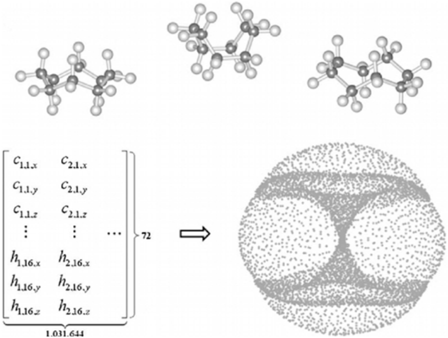
Jean-Daniel Boissonnat & Tamal Dey & Clément Maria

1 Introduction

Motivation

Given a set of geometric points, study the global features of the underlying space: components, holes, voids, etc.

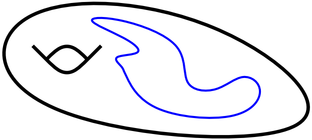
- Topological Data Analysis
- Machine Learning
- Vizualisation
- ...

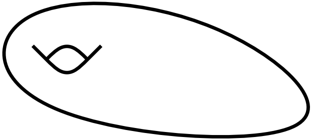


Conformation Space of the Cyclo-octane Molecule

Image: **[Martin, Thompson, Coutsias, Watson]**

Introduction to Homology





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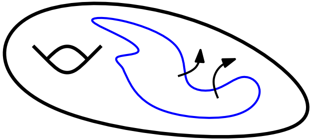

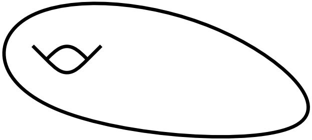

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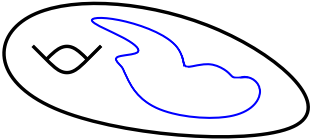

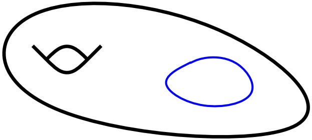

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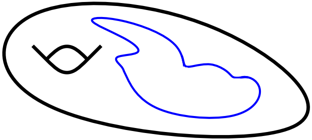

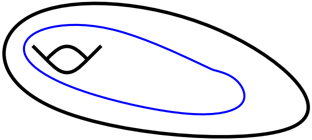

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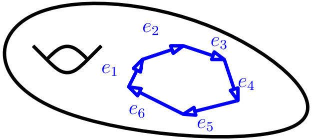
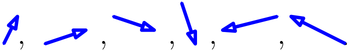


$\{e_1, \; e_2 \;, \; e_3 \;, e_4, \; e_5 \;, \; e_6 \;, \cdots \}$

$\mathbf{K}^p$: set of p-simplices of $\mathbf{K}$ $\{$  $, \cdots \}$

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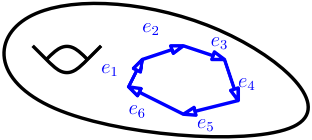
$e_1 + e_2 + e_3 + e_4 + e_5 + e_6$



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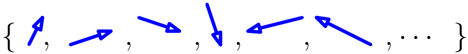

$\mathbf{C}_p$: Abelian group of formal sums of p-simplices with $\mathbb{Z}$-coefficients

$$\mathbf{C}_1 = (\{\sum_i k_i e_i\}, +)$$

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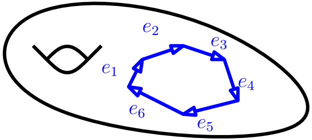

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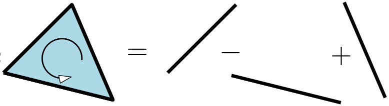
$\partial_p : \mathbf{C}_p \to \mathbf{C}_{p-1}$: Boundary operator

∂_2 

$$\partial_2 \; [a, b, c] = [ab] - [bc] + [ca]$$

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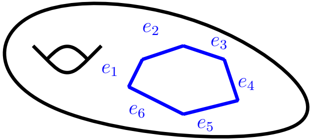

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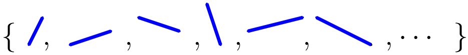
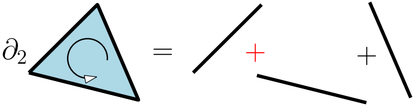


$\mathbf{C}_p$: Abelian group of formal sums of p-simplices with ~~$\mathbb{Z}$~~ $\mathbb{Z}_2$-coefficients

$\mathbf{C}_1 = (\{\sum_i k_i e_i\}, +)$ — $k_i \in \{0, 1\}$

$\partial_p : \mathbf{C}_p \to \mathbf{C}_{p-1}$: Boundary operator



$\partial_2 \, [a, b, c] = [ab] + [bc] + [ca]$

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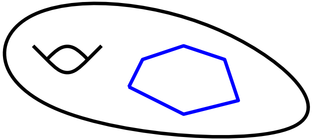

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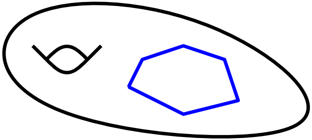

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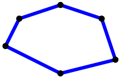
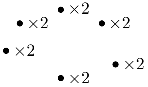
$\mathbf{Z}_p = \ker\, \partial_p$: the p-cycles

∂_1  $=$ 

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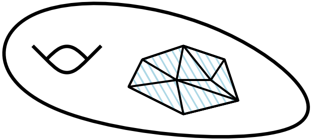

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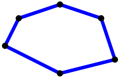
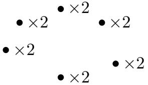
$\mathbf{Z}_p = \ker\ \partial_p$: the p-cycles ∂_1  $=$ 

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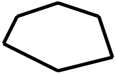
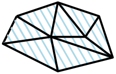
$\mathbf{B}_p = \mathrm{im}\ \partial_{p+1}$: the p-boundaries  $=$ ∂_2 

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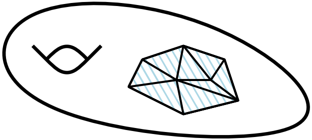

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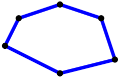
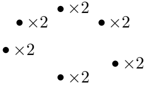
∂_1  $=$ 

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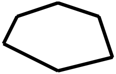
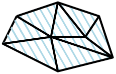
 $= \partial_2$ 

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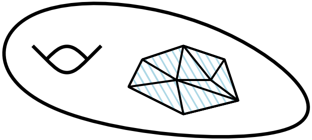

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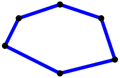
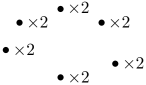
$\mathbf{Z}_p = \ker\ \partial_p$: the p-cycles ∂_1  $=$ 

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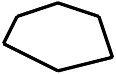
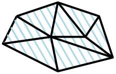
$\mathbf{B}_p = \mathrm{im}\ \partial_{p+1}$: the p-boundaries  $=$ ∂_2 

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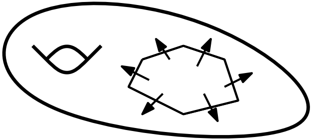

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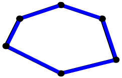
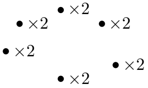
$\mathbf{Z}_p = \ker\ \partial_p$: the p-cycles ∂_1  $=$ 

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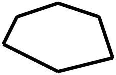
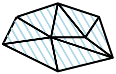
$\mathbf{B}_p = \mathrm{im}\ \partial_{p+1}$: the p-boundaries  $=$ ∂_2 

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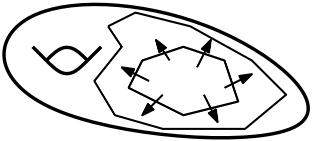

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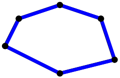
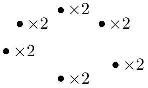
$\mathbf{Z}_p = \ker\ \partial_p$: the p-cycles ∂_1  $=$ 

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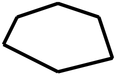
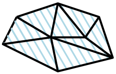
$\mathbf{B}_p = \mathrm{im}\ \partial_{p+1}$: the p-boundaries  $= \; \partial_2$ 

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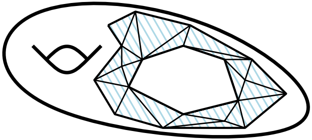

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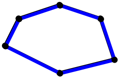
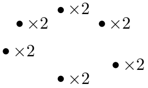
$\mathbf{Z}_p = \ker \, \partial_p$: the p-cycles $\quad \partial_1$  $=$ 

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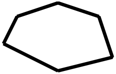
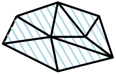
$\mathbf{B}_p = \mathrm{im} \, \partial_{p+1}$: the p-boundaries  $= \quad \partial_2$ 

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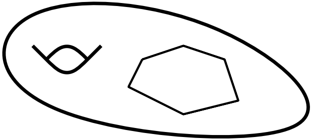

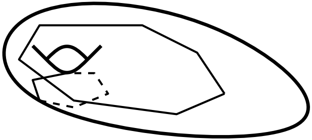

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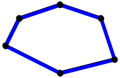
$\mathbf{Z}_p = \ker \partial_p$: the p-cycles ∂_1



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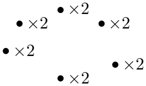

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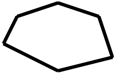
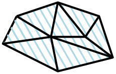


$= \partial_2$



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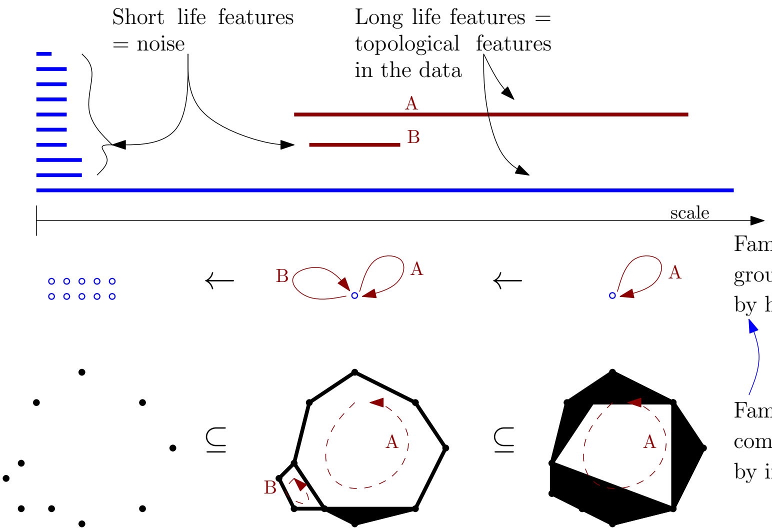
$\mathbf{H}_p = \mathbf{Z}_p / \mathbf{B}_p$: p^{th} homology group

Cohomology and Persistence



Family of cohomology groups linked by homomorphisms

Family of simplicial complexes linked by inclusions

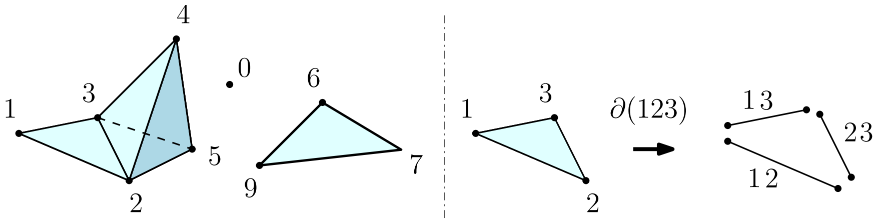
Simplicial Complex

Given a set $V = \{1 \cdots n\}$ of vertices, an **abstract simplicial complex** $\mathcal{K}$ on V is a family of subsets of vertices s.t.:

$$\forall \sigma \in \mathcal{K} : \tau \subseteq \sigma \Rightarrow \tau \in \mathcal{K}$$

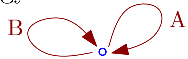
Such σ is called a simplex

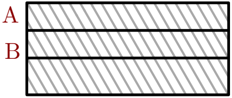
Boundary of a simplex σ: $\partial\sigma = \{\tau \subset \sigma | \dim(\sigma) = \dim(\tau) + 1\}$



Representation

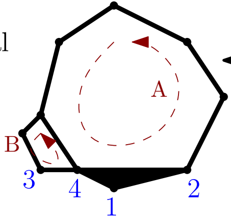
Cohomology group

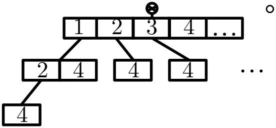




Matrix representation

Simplicial complex

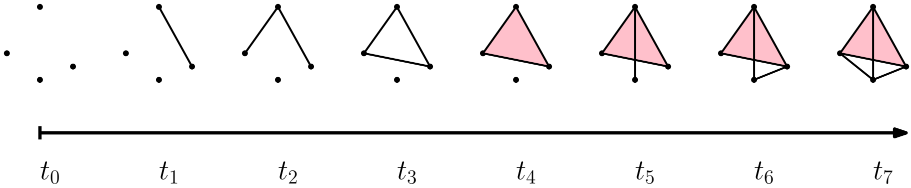




- Simplex tree [Boissonnat, M. '12]

2
Persistent Cohomology Algorithm

Cohomology Algorithm



$$\mathcal{K}_0 \subseteq \mathcal{K}_1 \subseteq \mathcal{K}_2 \subseteq \mathcal{K}_3 \subseteq \mathcal{K}_4 \subseteq \mathcal{K}_5 \subseteq \mathcal{K}_6 \subseteq \mathcal{K}_7$$

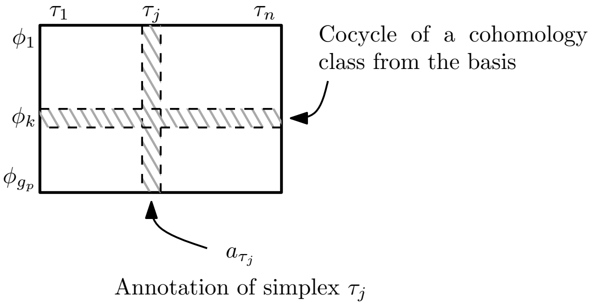
- Insert the simplices in the order of the filtration
- Update the cohomology groups accordingly

Cohomology Algorithm

[de Silva, Vejdemo-Johansson, Morozov '11],[Dey, Fan, Wang]

$\mathcal{K}_i$

$H^p(\mathcal{K}_i) = ([\phi_1], \cdots, [\phi_{g_p}])$

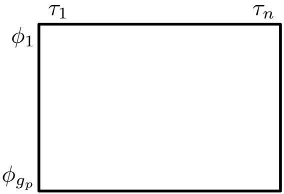


Cohomology Algorithm

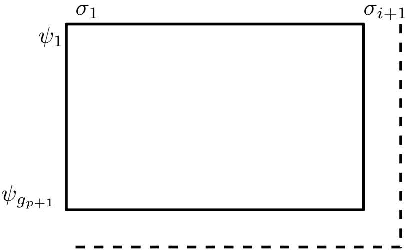
[de Silva, Vejdemo-Johansson, Morozov '11],[Dey, Fan, Wang]

$\mathcal{K}_i \hookrightarrow \mathcal{K}_i \cup \sigma_{i+1} = \mathcal{K}_{i+1}$ $\quad$ $H^*(\mathcal{K}_i) \to H^*(\mathcal{K}_{i+1})$

$H^p(\mathcal{K}_i)$



$H^{p+1}(\mathcal{K}_i)$

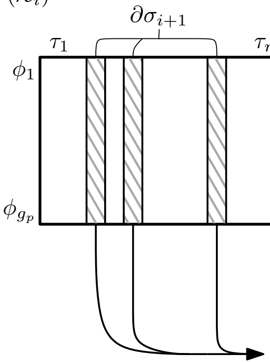


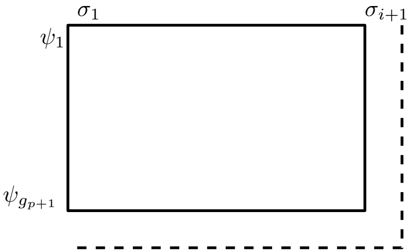
Cohomology Algorithm

[de Silva, Vejdemo-Johansson, Morozov '11],[Dey, Fan, Wang]

$\mathcal{K}_i \hookrightarrow \mathcal{K}_i \cup \sigma_{i+1} = \mathcal{K}_{i+1}$ $\qquad H^*(\mathcal{K}_i) \to H^*(\mathcal{K}_{i+1})$

$H^p(\mathcal{K}_i)$ $\qquad H^{p+1}(\mathcal{K}_i)$





$$a_{\partial\sigma_{i+1}} = \sum_{\tau \in \partial\sigma_{i+1}} (\pm 1)\, a_\tau$$

Cohomology Algorithm

[de Silva, Vejdemo-Johansson, Morozov '11],[Dey, Fan, Wang]

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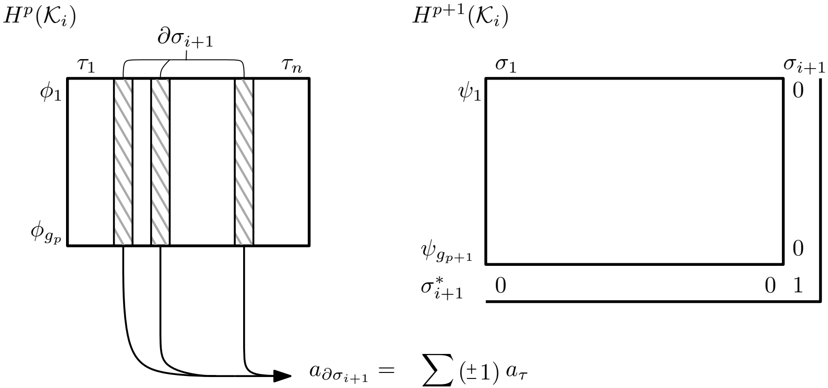


$$a_{\partial\sigma_{i+1}} = \sum_{\tau \in \partial\sigma_{i+1}} (\pm 1)\, a_\tau$$

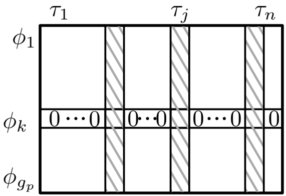
(i) $a_{\partial\sigma_{i+1}} = 0 \Rightarrow \sigma^*_{i+1}$ independent cocycle

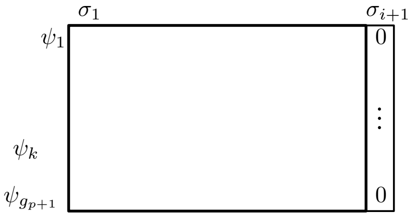
Cohomology Algorithm

[de Silva, Vejdemo-Johansson, Morozov '11],[Dey, Fan, Wang]

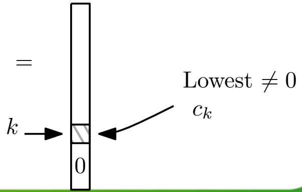
$\mathcal{K}_i \hookrightarrow \mathcal{K}_i \cup \sigma_{i+1} = \mathcal{K}_{i+1}$ $\qquad$ $H^*(\mathcal{K}_i) \to H^*(\mathcal{K}_{i+1})$

$H^p(\mathcal{K}_i)$ $\qquad$ $H^{p+1}(\mathcal{K}_i)$





$$a_{\partial\sigma_{i+1}} = \sum_{\tau \in \partial\sigma_{i+1}} (\pm 1)\, a_\tau \quad =$$



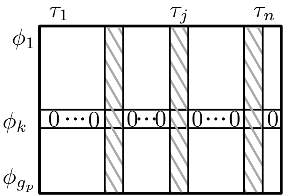
(ii) $a_{\partial\sigma_{i+1}} \neq 0 \Rightarrow$ Reduction

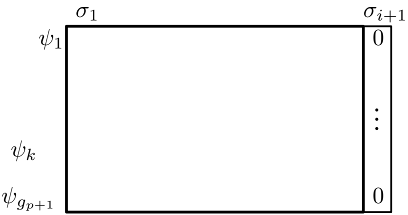
Cohomology Algorithm

[de Silva, Vejdemo-Johansson, Morozov '11],[Dey, Fan, Wang]

$\mathcal{K}_i \hookrightarrow \mathcal{K}_i \cup \sigma_{i+1} = \mathcal{K}_{i+1}$ $\qquad$ $H^*(\mathcal{K}_i) \to H^*(\mathcal{K}_{i+1})$

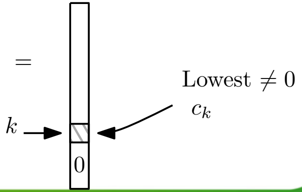
$H^p(\mathcal{K}_i)$ $\qquad$ $H^{p+1}(\mathcal{K}_i)$





$$a_{\tau_j} \leftarrow a_{\tau_j} - \frac{a_{\tau_j}[k]}{c_k} a_{\partial\sigma_{i+1}}$$

$$a_{\partial\sigma_{i+1}} = \sum_{\tau \in \partial\sigma_{i+1}} (\pm 1)\, a_\tau \quad =$$

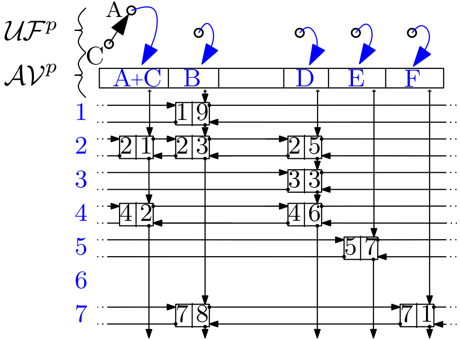


(ii) $a_{\partial\sigma_{i+1}} \neq 0 \Rightarrow$ Reduction

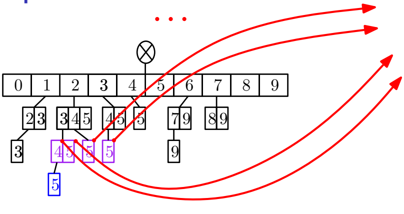
3
Compressed Annotation Matrix

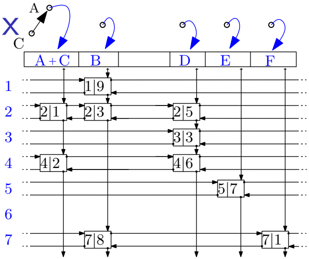
Compressed Annotation Matrix

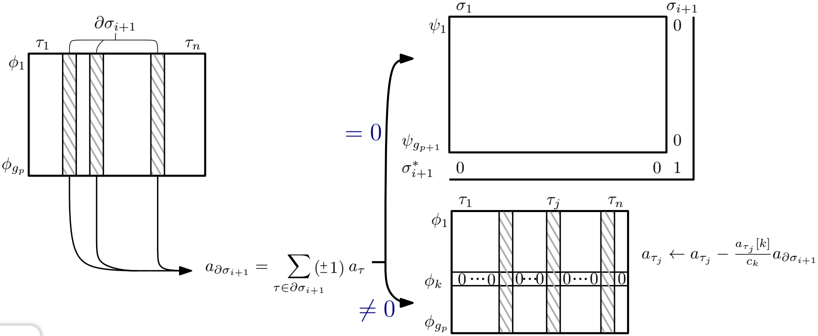
	A	B	C	D	E	F
1	0	9	0	0	0	0
2	1	3	1	5	0	0
3	0	0	0	3	0	0
4	2	0	2	6	0	0
5	0	0	0	0	7	0
6	0	0	0	0	0	0
7	0	8	0	0	0	1



Compressed Annotation Matrix

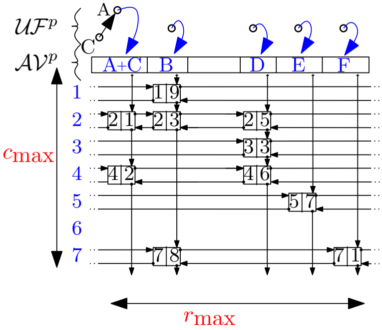






Complexity Analysis

- m number of simplices
- k dimension of the complex
- $c_{\max}$ longest column
- $r_{\max}$ longest row
- $\mathcal{C}_{\mathcal{AV}}$: operation in $\mathcal{AV}$
- $\mathcal{C}_{\partial}$: boundary computation of σ
- $\alpha(\cdot)$ inverse Ackermann function

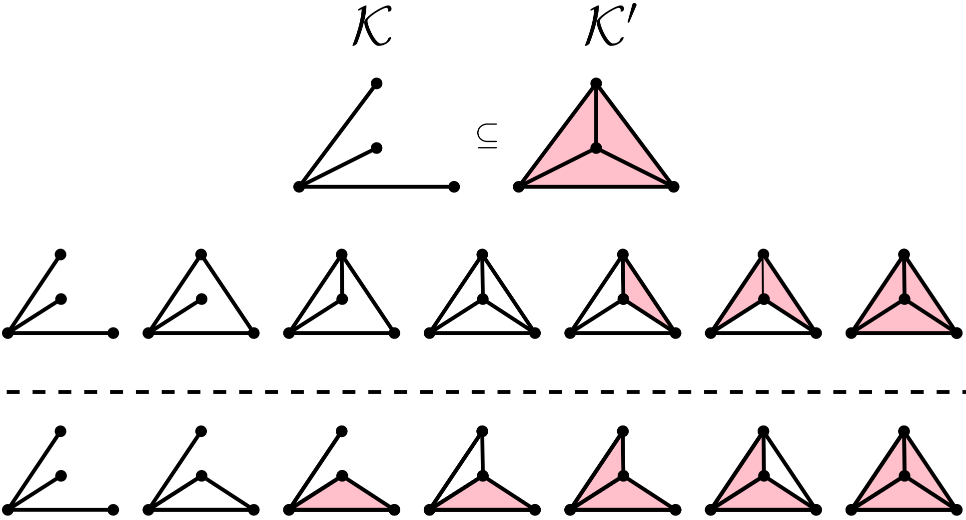


$$O\left(m \times [\mathcal{C}_{\partial} + k(\alpha(m) + c_{\max}) + r_{\max}(c_{\max} + \mathcal{C}_{\mathcal{AV}} + \alpha(m))]\right)$$

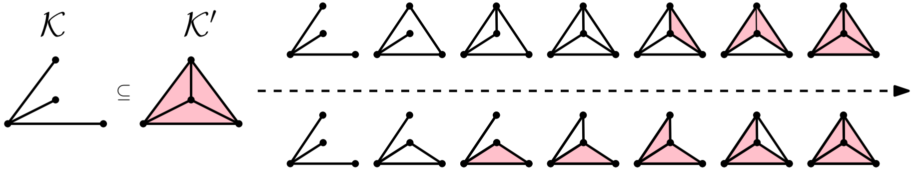
and simplifying a bit: $O(m\, c_{\max}(k + r_{\max}))$

4
Reordering the Simplices

Reordering Iso-Simplices

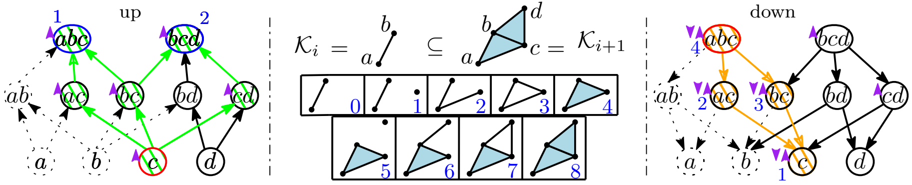


Reordering Iso-Simplices



Given a simplex σ

- Order its maximal cofaces "depth-first"
- Insert the maximal faces and their subfaces



5 Experiments

Timings

Data	Cpx	D	d	$\|\mathcal{K}\|$	DioCoH $\mathbb{Z}_2$	DioCoH $\mathbb{Z}_{11}$	PHAT$^\perp$ $\mathbb{Z}_2$	PHAT$^\perp$ $\mathbb{Z}_{11}$	PHAT $\mathbb{Z}_2$	PHAT $\mathbb{Z}_{11}$	CAM $\mathbb{Z}_2$	CAM $\mathbb{Z}_{11}$
Cy8	Rips	24	2	21×10^6	420	4822	44	—	5.3	—	6.4	6.5
S4	Rips	5	4	72×10^6	943	1026	95	—	3591	—	22.5	23.2
L57	Rips	—	3	34×10^6	239	240	35.2	—	972	—	9.3	9.5
Bro	Wit	25	?	3.2×10^6	807	T_∞	6.3	—	0.88	—	2.7	2.9
Kl	Wit	5	2	74×10^6	569	662	101	—	1785	—	19.7	19.9
L35	Wit	—	3	18×10^6	109	110	17.5	—	869	—	5.1	5.1
Bud	αSh	3	2	1.4×10^6	30.0	30.9	2.6	—	0.32	—	0.7	0.7
Nep	αSh	3	2	57×10^6	T_∞	T_∞	163	—	33	—	39.5	40.2

- DioCoH: Dionysus (persistent cohomology)
- PHAT$^\perp$: Phat (persistent cohomology)
- PHAT: Phat (persistent homology)

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CAM: Computational time $= 2.7 \times 10^{-7} \leq \cdot \leq 9.1 \times 10^{-7}$ seconds per simplex on all examples / all field coefficients

Statistics

Nep	$\lvert M \rvert$	$\#\mathbb{F}op.$
Compression	126057	84×10^6
¬Compression	574426	3860×10^6

Nep	average	maximum
c_{av}, c_{max}	0.79	18
r_{av}, r_{max}	1.02	18

Bro	time
Reordering	2.9 s.
¬Reordering	14.2 s.

Bro	$\mathbb{Z}_{11}$			$\mathbb{Q}$		
	M_{DS}	$a_{\partial\sigma}$	M_{op}	M_{DS}	$a_{\partial\sigma}$	M_{op}
	71%	19%	10%	67%	21%	12%

- Compression: matrix 4.5 times smaller
- Compression: 46 times less operations: ≤ 1.5 op. per simplex
- Complexity: $O(m\, c_{max}(k + r_{max}))$
- Reordering: 4.9 times faster
- $\mathbb{Z}_{11} \to \mathbb{Q}$: only 8% slower.

Question?

Thank you!