Cone Walk
Navigating a random Delaunay triangulation

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Planar Graph Navigation

**Input**  Planar graph \( G(V, E) \), \( p \in V \), \( q \in \mathbb{R}^2 \)

**Output**  NN of \( q \in V \), path \( \subseteq V \)
Point location in Geometric Data structures
(Delaunay Triangulation)

Want **pointer to face containing a point**
Point location in Geometric Data structures (Delaunay Triangulation)

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- Useful subroutine
Point location in Geometric Data structures (Delaunay Triangulation)

Want **pointer to face containing a point**

- Useful subroutine
  - $\Rightarrow$ Complexity bound useful elsewhere
Some assumptions...

- Smooth Convex Domain $\mathcal{D}$, $\text{Area}(\mathcal{D}) = 1$
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- **Smooth Convex Domain** $\mathcal{D}$, $\text{Area}(\mathcal{D}) = 1$

- $\Phi$ “$n$ uniformly random points” in $\mathcal{D}$

*Usually mean Poisson Process of rate 1*
Expected: $O(|pq| \sqrt{n})$ [Devroye et al.]

(Delaunay Triangulation)
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Greedy (Delaunay Triangulation)

Terminates for DT

$q$

$p$

(Delaunay Triangulation)
Terminates for DT
Trivial Bound: $O(n)$

In practice: really very efficient
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In practice: really very efficient

- Complicated dependance structure
- Non-Markovian
Wireless sensor networks.
Wireless sensor networks.
Face Routing $\Rightarrow$ Straight Walk
- Face Routing $\Rightarrow$ Straight Walk

- Greedy Routing $\Rightarrow$ Greedy Walk
New Results

Existance of algorithm with properties

- Deterministic
- $O(1)$-competitive
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Worst case on uniformly random input, $n \to \infty$

- $(\log^{3+\xi} n)$-memoryless
- $O(|pq|\sqrt{n} + \log^5 n)$ vertices accessed
- $O(|pq|\sqrt{n} \log \log n + \log^5 n)$ steps
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Bonus

- Stronger bound on degree of DT
Conjecture...

Existence of algorithm with properties

- Deterministic
- $O(1)$-competitive?

Worst case on uniformly random input, $n \to \infty$

- $(\log^{3+\epsilon} n)$ memoryless
- $O(|pq|\sqrt{n} + \log^1 n)$ vertices accessed
- $O(|pq|\sqrt{n} \log \log n + \log^1 n)$ steps

Bonus

- Stronger bound on degree of DT
Cone Walk
Number of Cones

- ‘Nearly’ iid rvs.
Number of Cones

- ‘Nearly’ iid rvs..
- Clever conditioning + Hoeffding type inequalities
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Number of Cones

- ‘Nearly’ iid rvs..
- Clever conditioning + Hoeffding type inequalities
- \( \Rightarrow O(|pq|\sqrt{n}) \) cones
Sites Accessed
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$E[\text{number of border points}] = \sum_i \int_{U_i} P(B_W(x))d\nu = O(|pq|\sqrt{n})$
Problem with border 1
Sites Accessed

Problem with border 2
Contributes $O(|pq| \sqrt{n})$
$E[\Delta_{\phi^*}] = \Theta\left(\frac{\log n}{\log \log n}\right)$ [Eppstein et al.]
Stronger Degree Bound in DT

Largest number of points in circle of this radius \((2 \log n)\)

Radius: \(O(\sqrt{\log n})\)

\[\mathbb{P}(\Delta_{\Phi^*} > \log n) \leq \frac{1}{n}\]
Stronger Degree Bound in DT

\[ \Pr(\Delta \Phi | \Phi^* > \log^3 n) \leq \frac{1}{n} \]
$$\mathbb{P}(\Delta \phi > \log^{2+\xi} n) < \exp(-\log^{1+\xi/4} n)$$

$$\xi > 0, \ n \text{ large enough}$$
Corollaries

- Memorylessness
- Algorithmic Complexity
Thanks
- No deterministic memoryless algo with constant competitiveness on arbitrary triangulation [Bose et al.]

- No competitive algorithm under link length for DT [Bose et al.]

- No algorithm better than random walk, for arbitrary convex subdivision [Devroye et al.]