

Cone Walk

Navigating a random Delaunay triangulation

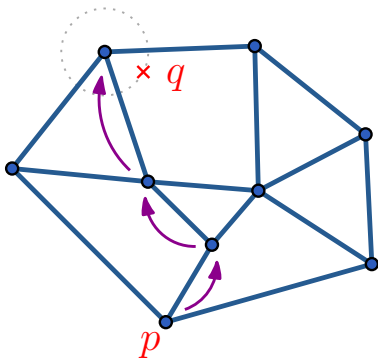
Nicolas Broutin, Olivier Devillers, Ross Hemsley

December 18, 2013

Planar Graph Navigation

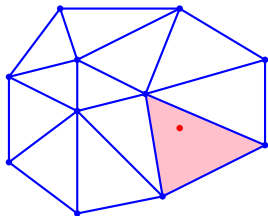
Input Planar graph $G(V, E)$, $p \in V$, $q \in \mathbb{R}^2$

Output NN of $q \in V$, path $\subseteq V$



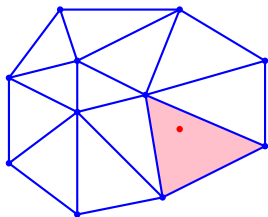
Point location in Geometric Data structures (Delaunay Triangulation)

Want **pointer to face containing a point**



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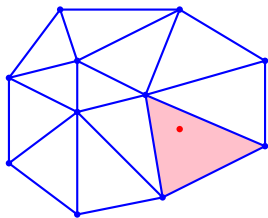
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- ▶ Useful subroutine

Point location in Geometric Data structures (Delaunay Triangulation)

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- ▶ \Rightarrow Complexity bound useful elsewhere

Some assumptions...

- ▶ Smooth Convex Domain \mathcal{D} , $\text{Area}(\mathcal{D}) = 1$

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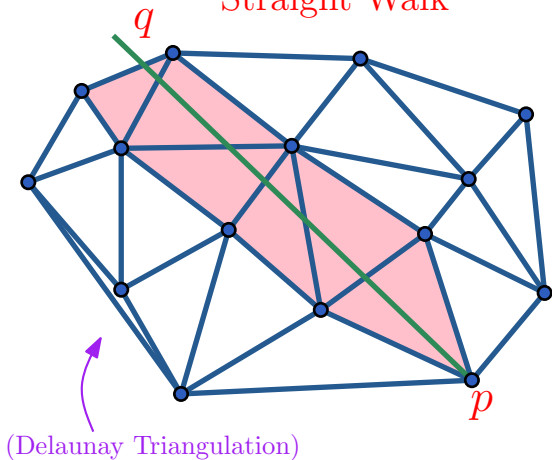
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- ▶ Φ “ n uniformly random points” in \mathcal{D}^*

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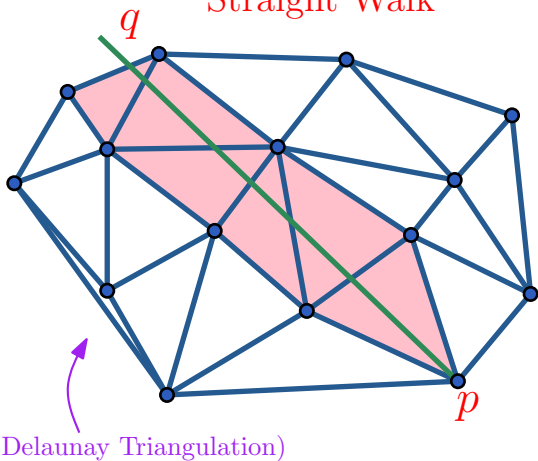
- ▶ Smooth Convex Domain \mathcal{D} , $\text{Area}(\mathcal{D}) = 1$
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*Usually mean Poisson Process of rate 1

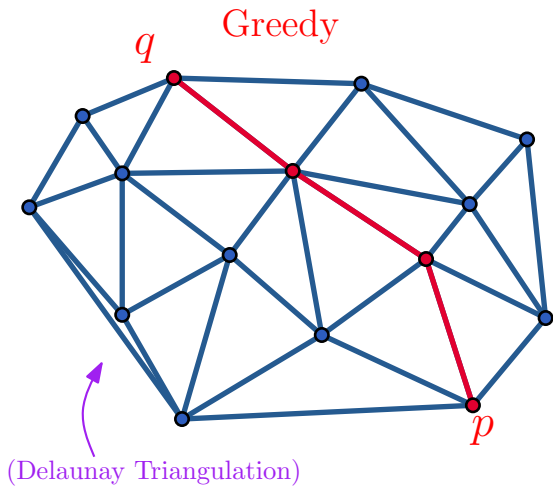
Straight Walk

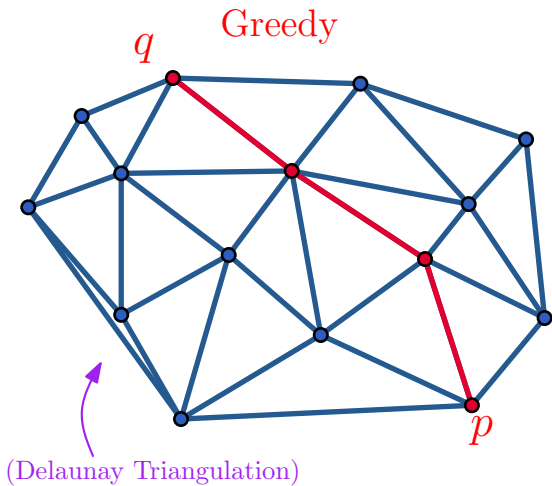


Straight Walk



Expected: $O(|pq|\sqrt{n})$ [Devroye et al.]





Terminates for DT

Trivial Bound: $O(n)$

In practice: really very efficient

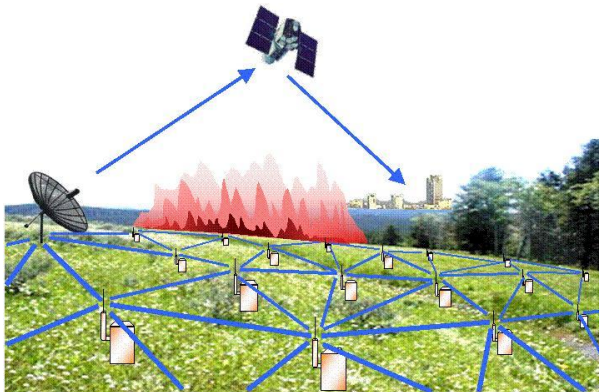
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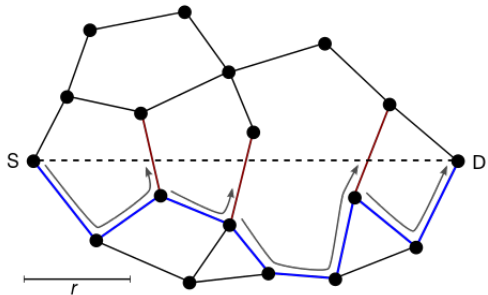
- ▶ Complicated dependance structure
- ▶ Non-Markovian

Wireless sensor networks.

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► Face Routing \Rightarrow Straight Walk



New Results

Existence of algorithm with properties

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- ▶ $(\log^{3+\xi} n)$ -memoryless
- ▶ $O(|pq|\sqrt{n} + \log^5 n)$ vertices accessed
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Bonus

- ▶ Stronger bound on degree of DT

Conjecture...

Existence of algorithm with properties

- ▶ Deterministic
- ▶ $O(1)$ -competitive?

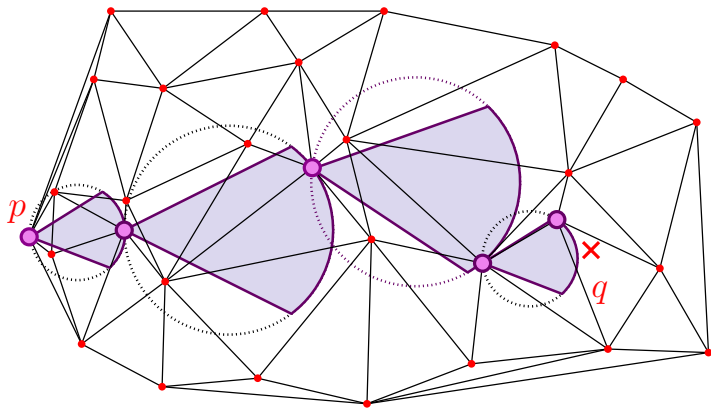
Worst case on uniformly random input, $n \rightarrow \infty$

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Cone Walk



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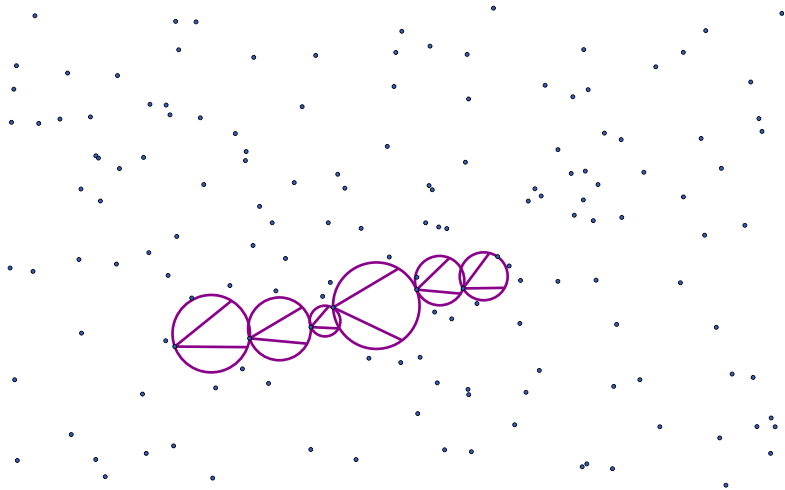
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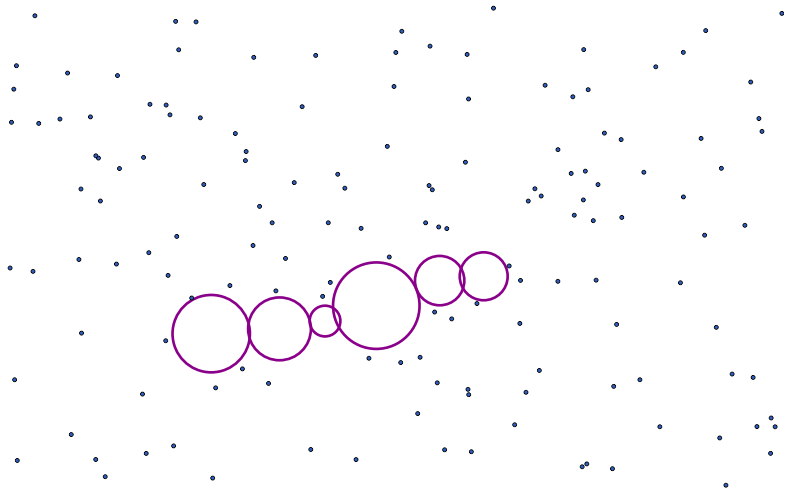
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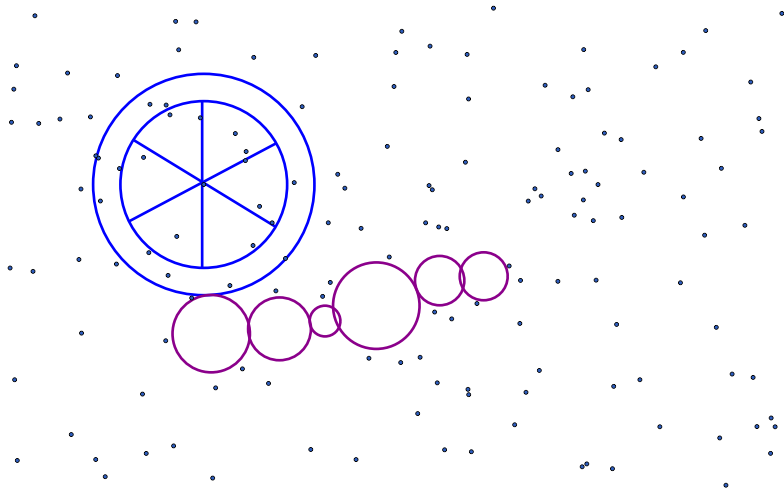
Sites Accessed



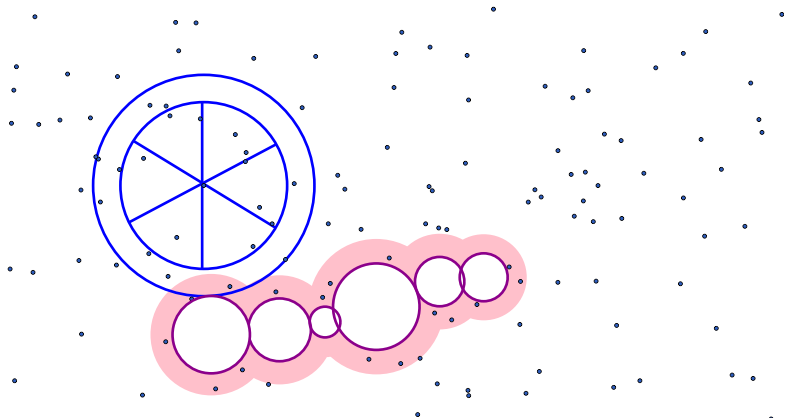
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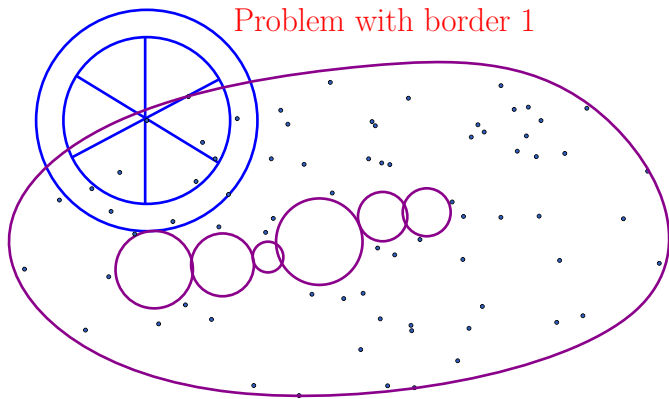
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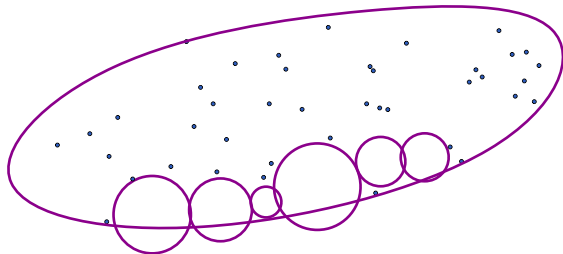
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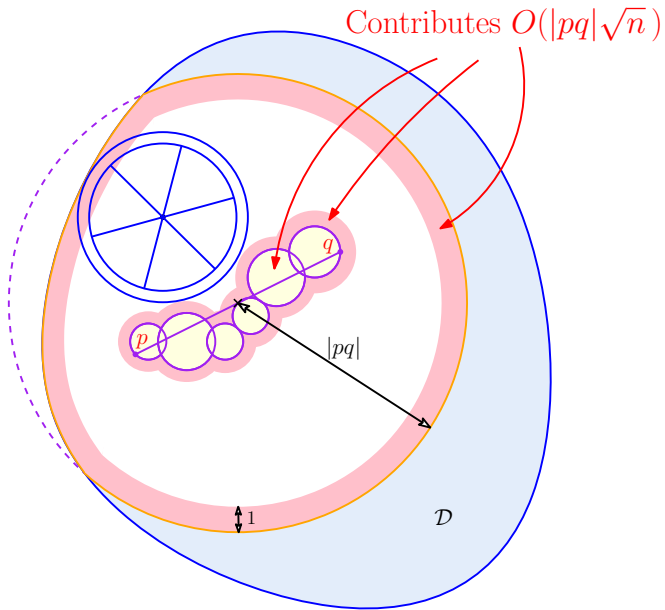


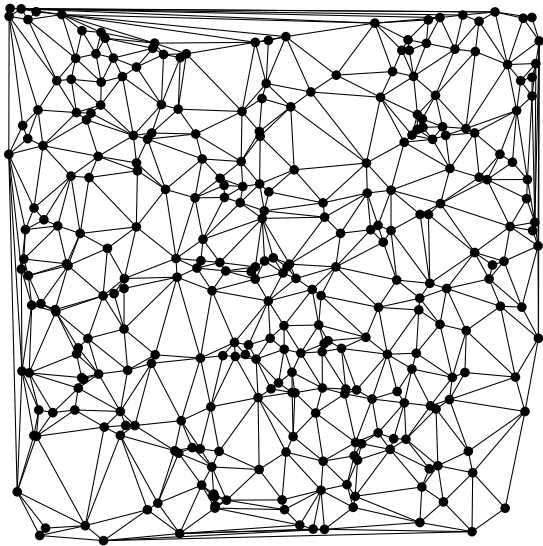
$$\mathbb{E}[\text{number of border points}] = \sum_i \int_{U_i} \mathbb{P}(B_W(x)) d\nu = O(|pq|\sqrt{n})$$

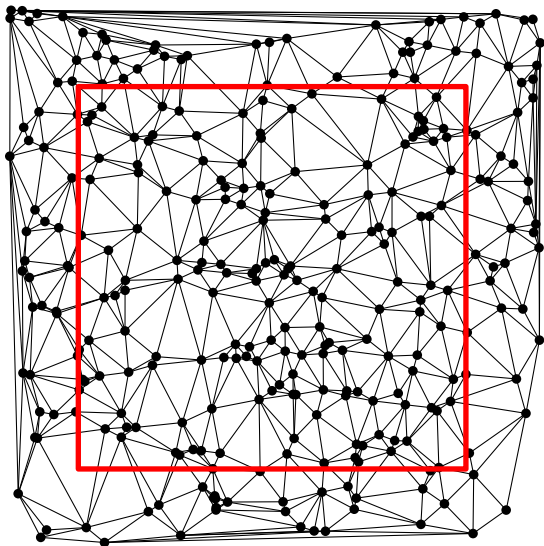


Problem with border 2



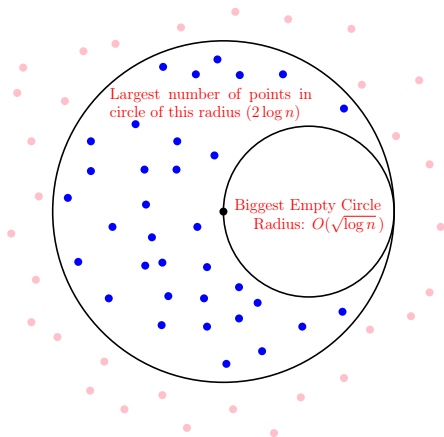






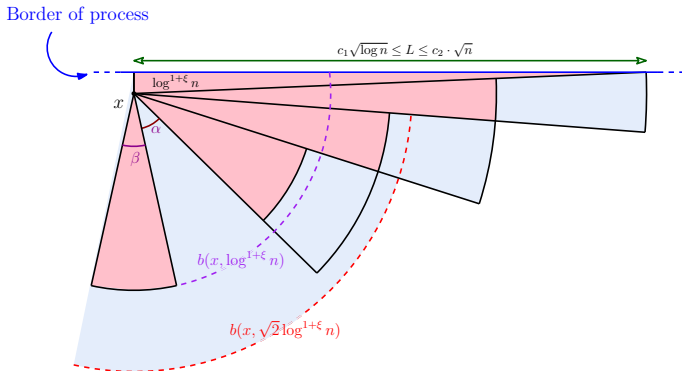
$$\mathbb{E}[\Delta_{\Phi^*}] = \Theta\left(\frac{\log n}{\log \log n}\right) \text{ [Eppstein et al.]}$$

Stronger Degree Bound in DT



$$\mathbb{P}(\Delta_{\phi^*} > \log n) \leq \frac{1}{n}$$

Stronger Degree Bound in DT



$$\mathbb{P}(\Delta_{\phi \setminus \phi^*} > \log^3 n) \leq \frac{1}{n}$$

$$\mathbb{P}(\Delta_\phi > \log^{2+\xi} n) < \exp(-\log^{1+\xi/4} n)$$

$\xi > 0, n$ large enough

Corollaries

- ▶ Memorylessness
- ▶ Algorithmic Complexity

Thanks

- ▶ No deterministic memoryless algo with constant competitiveness on arbitrary triangulation [Bose et al.]
- ▶ No competitive algorithm under link length for DT [Bose et al.]
- ▶ No algorithm better than random walk, for arbitrary convex subdivision [Devroye et al.]