Cone Walk

Navigating a random Delaunay triangulation

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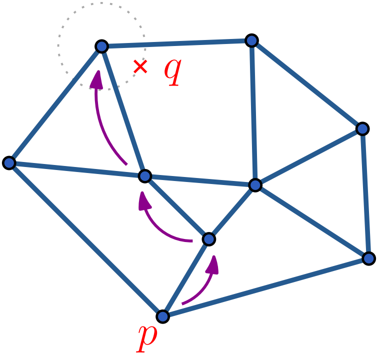
Planar Graph Navigation

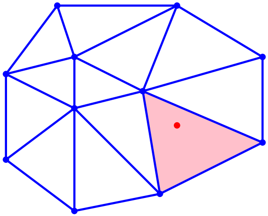
Input Planar graph $G(V, E)$, $p \in V$, $q \in \mathbb{R}^2$

Output NN of $q \in V$, path $\subseteq V$



Point location in Geometric Data structures
(Delaunay Triangulation)

Want pointer to face containing a point



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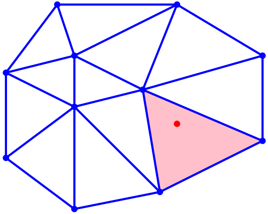

- Useful subroutine

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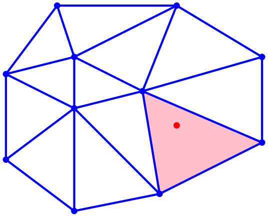

- Useful subroutine
- $\Rightarrow$ Complexity bound useful elsewhere

Some assumptions...

- Smooth Convex Domain $\mathcal{D}$, $\text{Area}(\mathcal{D}) = 1$

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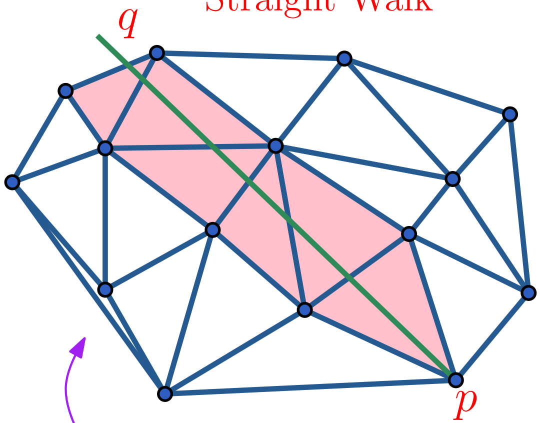
*Usually mean Poisson Process of rate 1

Straight Walk



(Delaunay Triangulation)

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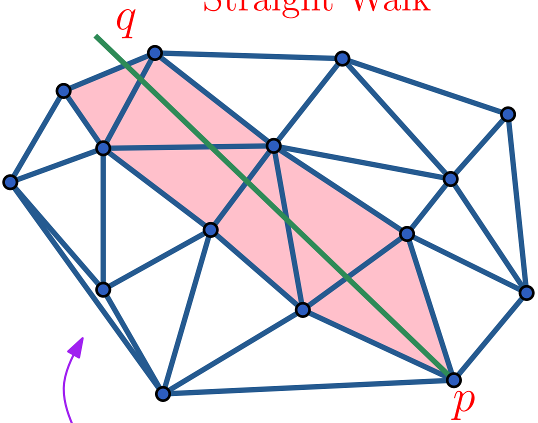
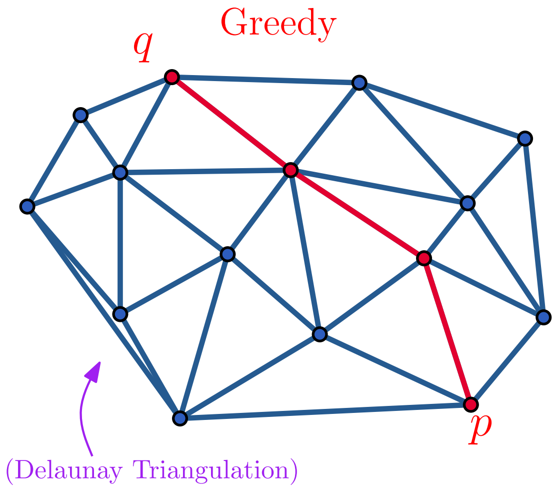
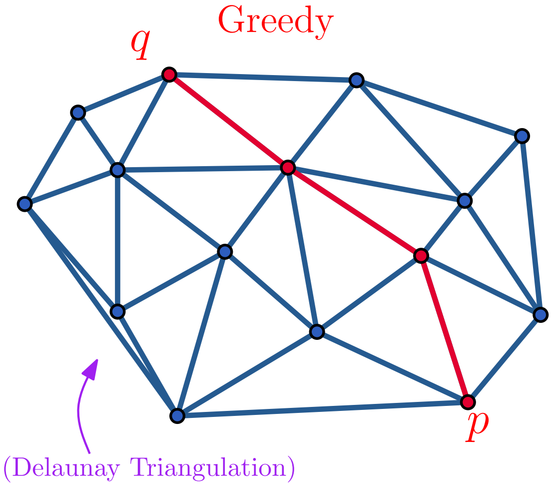


(Delaunay Triangulation)

Expected: $O(|pq|\sqrt{n})$ [Devroye et al.]


Greedy
q
p
(Delaunay Triangulation)



Terminates for DT

Trivial Bound: $O(n)$

In practice: really very efficient

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- Complicated dependance structure
- Non-Markovian

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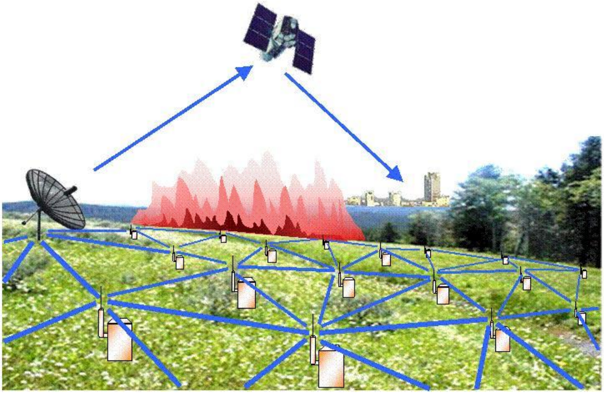
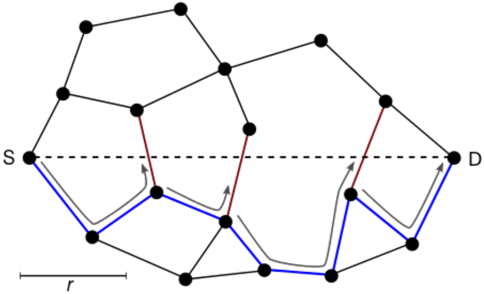
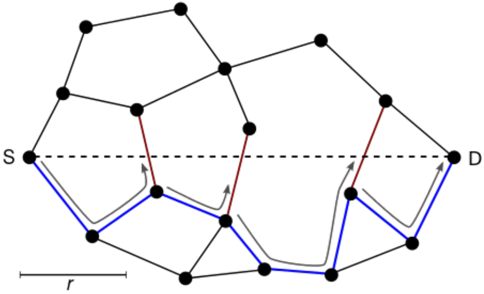


- Face Routing ⇒ Straight Walk



- Face Routing $\Rightarrow$ Straight Walk



- Greedy Routing $\Rightarrow$ Greedy Walk

New Results

Existance of algorithm with properties

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Bonus

- Stronger bound on degree of DT

Conjecture...

Existance of algorithm with properties

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- $O(1)$-competitive?

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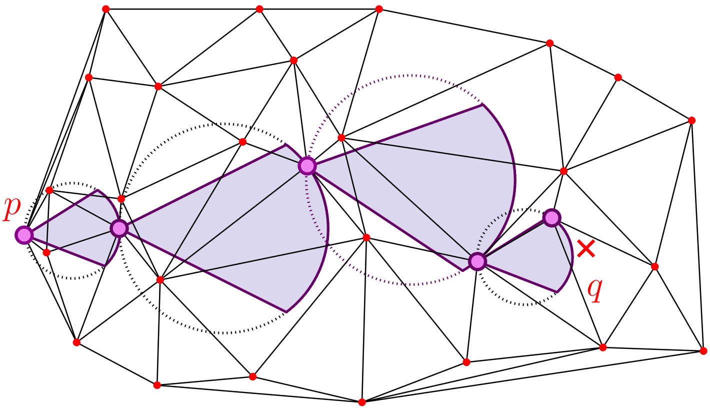
Cone Walk



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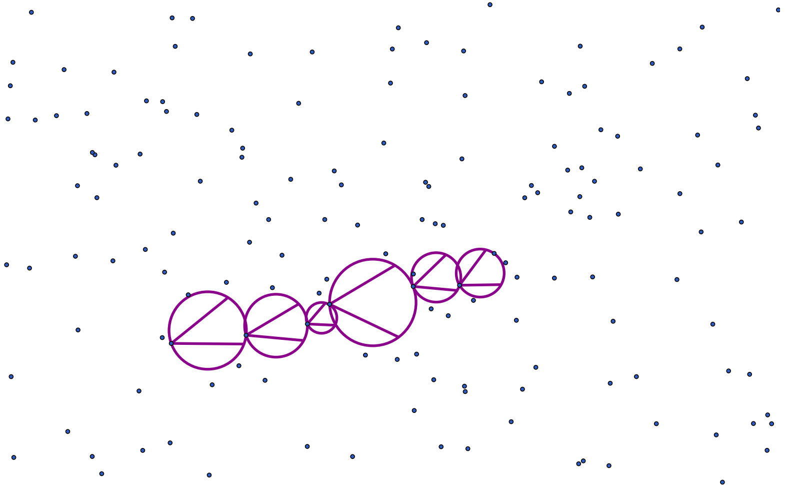

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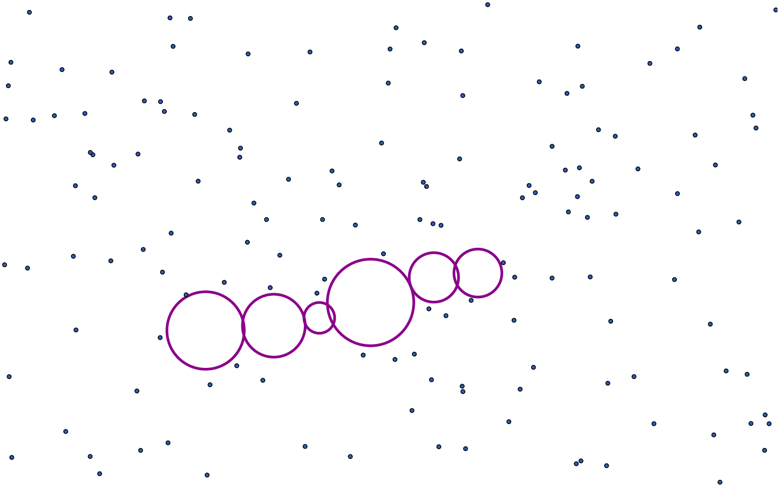

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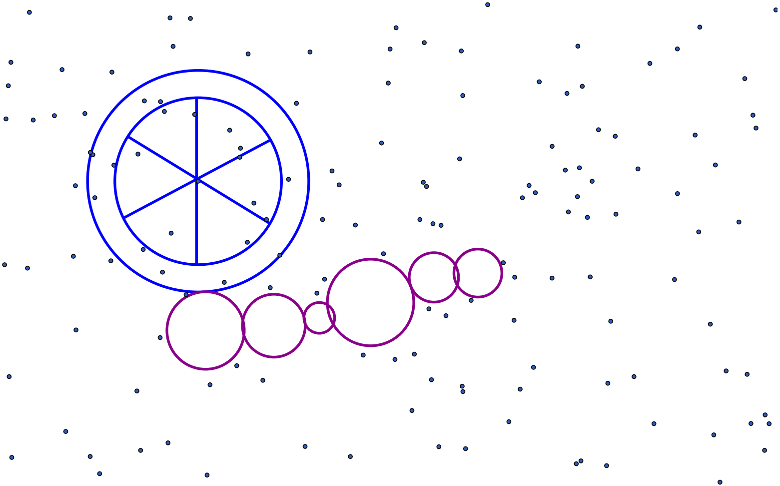

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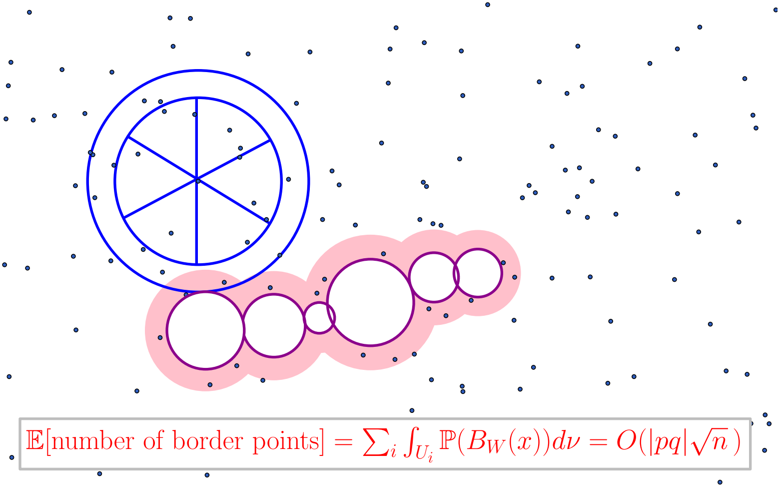

$$\mathbb{E}[\text{number of border points}] = \sum_i \int_{U_i} \mathbb{P}(B_W(x))d\nu = O(|pq|\sqrt{n}\,)$$

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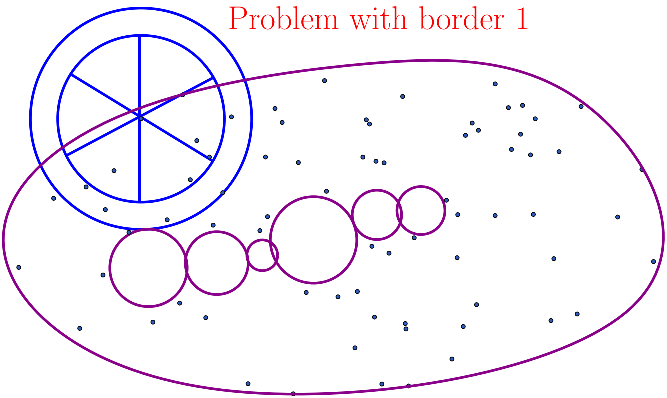

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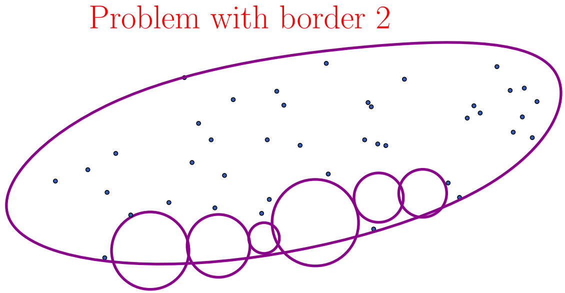
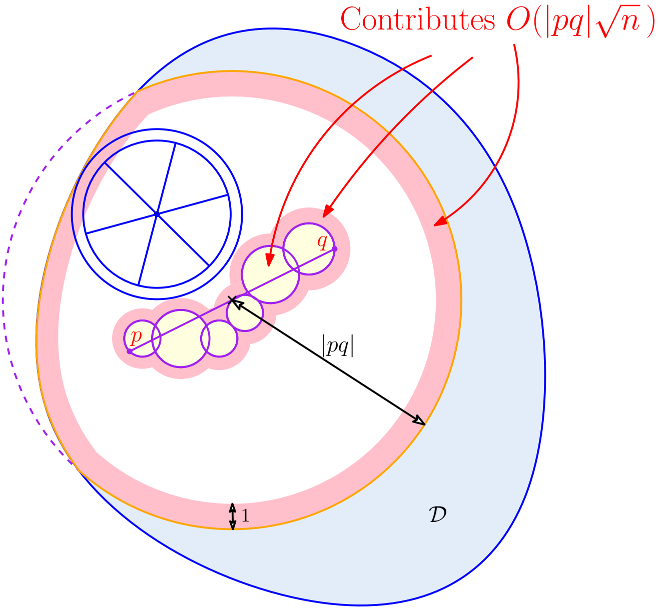
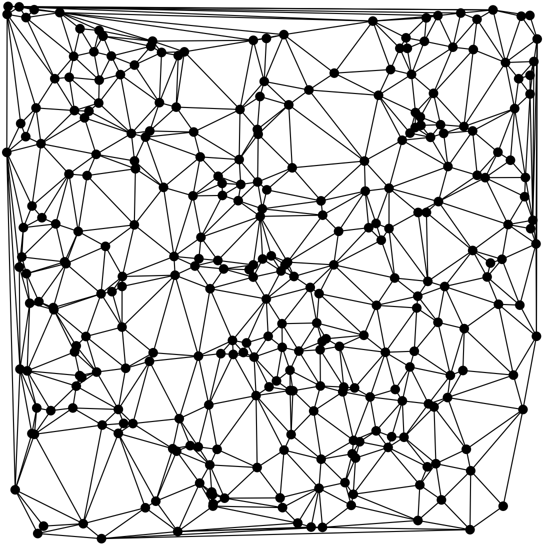
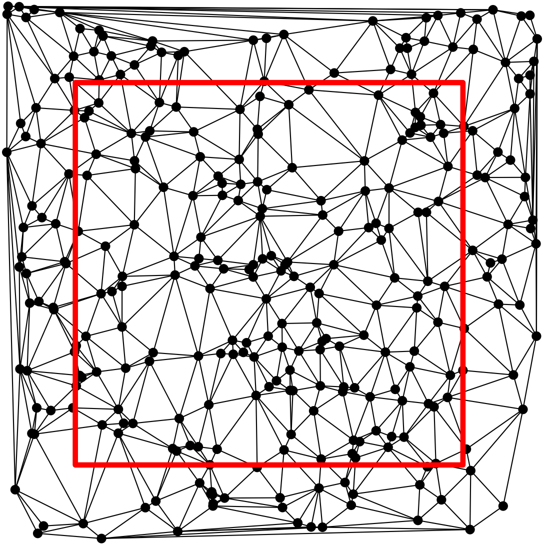




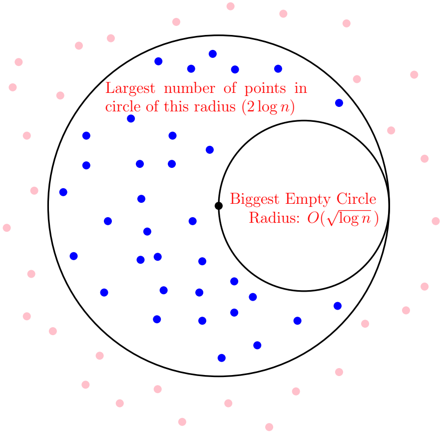
Contributes $O(|pq|\sqrt{n})$
q
p
$|pq|$
1
$\mathcal{D}$





$\mathbb{E}[\Delta_{\Phi^\star}] = \Theta(\frac{\log n}{\log \log n})$ [Eppstein et al.]

Stronger Degree Bound in DT



$$\mathbb{P}(\Delta_{\phi^\star} > \log n) \leq \frac{1}{n}$$

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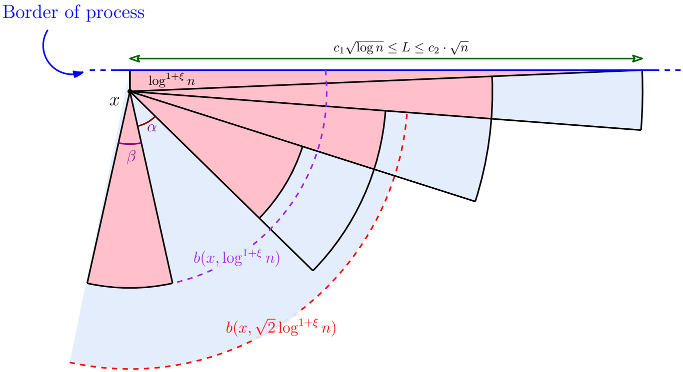

$$\mathbb{P}(\Delta_{\Phi\setminus\Phi^\star} > \log^3 n) \leq \frac{1}{n}$$

$$\mathbb{P}(\Delta_\Phi > \log^{2+\xi} n) < \exp(-\log^{1+\xi/4} n)$$

$\xi > 0$, n large enough

Corollaries

- Memorylessness
- Algorithmic Complexity

Thanks

- No deterministic memoryless algo with constant competitiveness on arbitrary triangulation [Bose et al.]

- No competitive algorithm under link length for DT [Bose et al.]

- No algorithm better than random walk, for arbitrary convex subdivision [Devroye et al.]