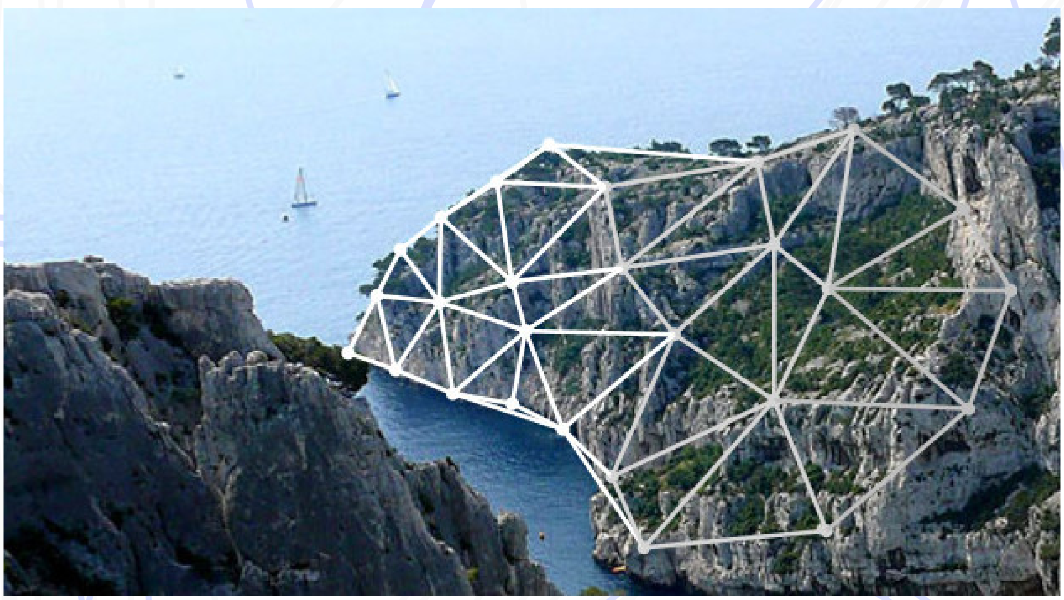
Journées de géométrie algorithmique 2013

CIRM, Luminy

Dec. 16–20, 2013



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LABORATOIRE JEAN KUNTZMANN

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 am		**Clément Maria** Multi-Field Persistent Homology		**Raphaëlle Chaine** Practical Reduction of Edge Flip Sequences in Two-Dimensional Triangulations	**Stephane Gaubert** Tropical convexity and its applications to zero-sum games
9:30 am	**David Cohen-Steiner** Improved bounds on higher eigenvalues of graphs	**Arnaud de Mesmay** Discrete systolic geometry and decompositions of triangulated surfaces	**Gabriel Peyré** Theoretical guarantees for variational regularization	**Julie Digne** Self-similarity for accurate compression of point sampled surface	
10:00 am	**Regis Straubhaar** Numerical optimization of Dirichlet-Laplace eigenvalues	**Anatolii Kostrygin** Periodic planar straight-frame graph drawings with polynomial resolution		**Viorica Patraucean** Mirror-symmetry in images and 3D shapes	
10:30 am					**Boris Thibert** Minkowski-type problems and optimal transport
11:00 am	**Christian Sohler** Coreset and sampling approaches for the analysis of very large data sets	**Christian Sohler** Coreset and sampling approaches for the analysis of very large data sets	**Michael Farber** Geometry and Topology of Random Spaces	**Michael Farber** Geometry and Topology of Random Spaces	**Rémi Thomasse** A convex body with chaotic random convex hull
11:30 am					**Quentin Mérigot** On the reconstruction of convex sets from random normal measurements
12:00 am					
12:30 am	**Lunch** Lunch	**Lunch** Lunch	**Lunch** Lunch	**Lunch** Lunch	**Lunch** Lunch
1:00 pm					

	Monday	Tuesday	Wednesday	Thursday	Friday
4:00 pm					
4:30 pm	**Gabriel Peyré** Sparse regularization of inverse problems	**Gabriel Peyré** First order optimization with proximal splitting		**Stephane Gaubert** Tropical convexity and its applications to zero-sum games	
5:00 pm			**Stephane Gaubert** Tropical convexity and its applications to zero-sum games		
5:30 pm					
6:00 pm	**Clément Maria** The Compressed Annotation Matrix: an Efficient Data Structure for Persistent Cohomology	**Thomas Bonis** Topological pooling for the bag of words model		**Benjamin Burton** Exploring parameterised complexity in computational topology	
6:30 pm	**Louis Cuel** Normals, curvature and sharp feature estimation using order-k Voronoi covariance	**Manish Mandad** TBA	**Ross Hemsley** Analysis for a walking algorithm on the Poisson Delaunay Triangulation		
7:00 pm	**Mickaël Buchet** Analyse de champs scalaires avec bruit aberrant	**Bertrand Michel** Convergence rates for persistence diagrams in topological data analysis	**Mael Rouxel-Labbé** Anisotropic mesh generation		
7:30 pm	**Dinner** Dinner	**Dinner** Dinner	**Dinner** Dinner	**Special dinner** Bouillabaise	
8:00 pm					
8:30 pm			**Business meeting**		
9:00 pm					

Abstracts of invited talks and mini-courses

Exploring parameterised complexity in computational topology

Benjamin Burton, The University of Queensland, Australia

Abstract. Topological decision problems in three dimensions are notoriously difficult: the mere existence of an algorithm is often a major result, and many important algorithms have yet to be studied from a complexity viewpoint. Even "simple" problems, such as unknot recognition (testing whether a loop of string is knotted), or 3-sphere recognition (testing whether a triangulated 3-manifold is topologically trivial), have best-known algorithms that are worst-case exponential time. In practice, however, some of these algorithms run surprisingly well in experimental settings, and we discuss some reasons why parameterised complexity now looks to be the "right" tool to explain this behaviour. We also present some initial forays into the parameterised complexity of topological problems, including both fixed-parameter-tractability and $W[P]$-hardness results.

Geometry and topology of large random spaces and groups

Michael Farber, University of Warwick

Abstract. I will discuss several probabilistic models producing simplicial complexes, manifolds and discrete groups. Random simplicial complexes are high dimensional analogues of random graphs and can be used for studying the behaviour of large systems or networks depending on many random parameters. We are interested in properties of random spaces which are satisfied with probability tending to one. Using probabilistic models one may also test probabilistically the validity of open topological questions such as the Whitehead and the Eilenberg–Ganea conjectures.

Tropical convexity and its applications to zero-sum games

Stéphane Gaubert, École polytechnique

Abstract. Tropical structures arise when the set of real numbers is equipped with the addition $(a,b) \mapsto \max(a,b)$ and the product $(a,b) \mapsto a+b$. We shall give here a guided tour of these structures, with emphasis on tropical convexity and its relation with optimal control, zero-sum games, and metric geometry.

Tropical convex sets are limits of classical convex sets by taking "log-glasses", or alternatively, as images by a non-archimedean valuation of convex sets over ordered fields (Puiseux or Hahn series). Many of the classical results of convex or functional analysis, including the theorems of Hahn-Banach, Krein-Milman, Helly, and Carathéodory, have tropical analogues, sometimes with significant variations. In particular, the familiar Euclidean metric is replaced by Hilbert's projective metric. However, the combinatorics of tropical polyhedra is still only partly understood. For instance, the classical bound of McMullen for the number of extreme points of a polyhedron defined by a collection of inequalities is not known to be tightin the tropical world.

A specific motivation to look at tropical convexity arises from mean payoff (ergodic) zero-sum two player problems. The sets of sub or super fixed points of Shapley operators (certificates that the game is winning for one player) turn out to be tropical convex sets. It follows that deciding whether a given state is winning, in a deterministic game, which is one of the problems in NP $\cap$ co-NP not known to be in P, is equivalent to the feasibility problem in tropical linear programming. Tropical linear programming appears to be closely related to linear programming over fields of formal series, and also to linear programming with large coefficients.

Infinite dimensional tropical convex sets appear to be interest, in particular, in optimal control, and in the construction of boundaries of metric spaces. Some recent applications to the reduction of the curse of dimensionality in optimal control will finally be presented.

This minilecture is based on works with different coauthors, M. Akian, X. Allamigeon, P. Benchimol, E. Goubault, A. Guterman, R. Katz, M. Joswig, G. Vigeral, and C. Walsh, as well as works of M. Develin and B. Sturmfels, W. Briec and C. Horvath, W. McEneaney, and Z. Qu.

Low complexity regularization of inverse problems

Gabriel Peyré, CNRS - Université Paris-Dauphine

Abstract. In this short course, I will review in an unified way the structural properties of a large class of convex regularizers for linear inverse problems. These penalty functionals are crucial to force the regularized solution to conform to some notion of simplicity/low complexity. Classical priors of this kind includes sparsity, piecewise regularity and low-rank. These are natural assumptions for many applications, ranging from medical imaging to machine learning. I will discuss theoretical assessments of the recovery performances of these regularizations, which include robustness to noise in the measurements. This encompasses several previous works focused both on model estimation, L^2 stability, and it shares links with the compressed sensing literature when considering randomized measurements. I will also discuss the numerical computation of the solution to the corresponding convex programs using first order proximal methods.

Coreset and sampling approaches for the analysis of very large data sets

Christian Sohler, TU Dortmund

Abstract. In my talk I will discuss two related approaches to analyze very large data sets. For unstructured data, i.e. point sets, I will introduce the concept of coresets using the example of k-means clustering and discuss different techniques to develop them. Then I will explain how coresets can be used to obtain streaming algorithms and distributed algorithms. In the second part of my talk I will focus on semi-structured data, i.e. graphs. I will introduce the framework of Property Testing that allows to study sampling processes to obtain information about global properties of graphs by random sampling. In my talk I will focus on the recent developments in Property Testing for bounded degree graphs that finally lead to a local vs. global theorem for planar graphs: Informally speaking, any bounded degree planar graph is specified upto eps n edges by its local structure, i.e. the distribution of constant sized BFS neighborhoods.

Abstracts of contributed talks

Topological pooling

Thomas Bonis, INRIA Geometrica

Abstract. We propose a new pooling method based on persistent homology for the bag of features approach for object recognition. We then compare this method to the standard method on various images datasets as well as on a 3D shapes dataset.

Analyse de champs scalaires avec bruit aberrant

Mickaël Buchet, INRIA Geometrica

Abstract. Étant donné une fonction f définie sur une sous-variété d'un espace euclidien, nous souhaitons étudier la structure de f en ne connaissant sa valeur qu'en un nombre fini de points. De plus, ces valeurs sont susceptibles d'être bruitées. Les méthodes existantes permettent d'approximer le diagramme de persistance de f lorsque le bruit est borné. Cependant, elles échouent lorsque les données contiennent des valeurs aberrantes.

Nous présentons un algorithme permettant de traiter ces valeurs sans les connaître a priori. Il permet d'approximer la fonction f et son diagramme de persistance d'une manière garantie théoriquement.

Practical reduction of edge flip sequences in two-dimensional triangulations

Raphaelle Chaine, LIRIS, Université Lyon 1

Abstract. The development of laser scanning techniques has popularized the representation of 3D shapes by triangular meshes with a large number of vertices. Compression techniques dedicated to such meshes have emerged, which exploit the idea that the connectivity of a dense mesh does not deviate much from the connectivity that can be constructed automatically from the vertex positions. The edge flip is one of the tools that can encode the differences between two meshes, and it is important to control the length of a sequence of flips that transform one triangulation into another. This paper provides a practical solution to this problem. Indeed, the problem of determining a minimal sequence of edge flips between two triangulations is NP-complete for some types of triangulations including manifold triangulations of surfaces, so that it is necessary to develop heuristics. Moreover, it is sometimes difficult to establish a first sequence of flips between two meshes, and we propose a solution based on the reduction of an existing sequence. The new approach we propose is founded on the assignment of labels to identify the edges, with a property of label transfer during a flip. All the operations are performed directly on the sequence of labels denoting the edges to be flipped, almost regardless of the underlying surface, since only local temporary connectivity is involved:

- Given an initial triangulation T, and a sequence of edge flips Φ, we introduce three simple moves to generate new sequences Γ such that Φ and Γ transform T into a similar triangulation, up to a permutation of its labels.

- On the basis of these three moves, we propose an efficient algorithm to reduce the input sequence of flips Φ, while ensuring a local optimization of the sequence length.

This algorithm is polynomial and can be used in a purely combinatorial setting as well as in a geometric setting. The effectiveness of our approach is illustrated by practical benchmarks and comments. Joint work with Jeremy Espinas and Pierre-Marie Gandoin.

Improved bounds on higher eigenvalues of graphs

David Cohen-Steiner, INRIA Geometrica

Abstract. The eigenvalues of a graph are intimately related to the existence of good partitions of the graph. We will show that recent eigenvalue bounds for Riemannian surfaces lead to improved bounds for graphs as a function of their genus. Our approach is based on a variant of Burger's comparison principle which is of independant interest. Joint work with Omid Amini.

Normals, curvature and sharp feature estimation using order-k Voronoi covariance

Louis Cuel, Université de Grenoble - Université de Chambéry

Abstract. We present a robust method to estimate normals, curvature directions and sharp features from an unorganized points cloud approximating hypersurace in $\mathbb{R}^d$. Using the k-order Voronoi diagram which contains geometrical informations, we define a covariance matrix for every point of the point cloud. We offer theorical garantees of the covariance matrix calculated without a Hausdorff hypothesis, which allows the presence of outliers. Experimentations confirm theorical results and show that our method gives a good sharp features estimation of a surface in $\mathbb{R}^3$.

Self-similarity for accurate compression of point sampled surface

Julie Digne, CNRS - Université Lyon 1

Abstract. Most surfaces, be it from a fine-art artifact or a mechanical object, are characterized by a strong self-similarity. This property finds its source in the natural structures of objects but also in the fabrication processes: regularity of the sculpting technique, or machine tool. In this paper, we propose to exploit the self-similarity of the underlying shapes for compressing point cloud surfaces which can contain millions of points at a very high precision. Our approach locally resamples the point cloud in order to highlight the self-similarity of the shape, while remaining consistent with the original shape and the scanner precision. It then uses this self-similarity to create an ad hoc dictionary on which the local neighborhoods will be sparsely represented, thus allowing for a light-weight representation of the total surface. We demonstrate the validity of our approach on several point clouds from fine-arts and mechanical objects, as well as a urban scene. In addition, we show that our approach also achieves a filtering of noise whose magnitude is smaller than the scanner precision.

Cone Walk: Navigating a random Delaunay triangulation

Ross Hemsley, INRIA Geometrica

Abstract. Planar graph navigation is a problem with significant implications in both point location and routing in networks. However, very little analysis is available for algorithms solving this problem under distribution hypotheses for the input. We propose and analyse a new deterministic planar navigation algorithm with constant competitiveness which follows vertex adjacencies in the Delaunay triangulation, giving bounds on the worst case behaviour for any possible instance of the algorithm applied to a uniformly distributed set of points in a smooth convex domain.

Periodic planar straight-frame graph drawings with polynomial resolution

Anatolii Kostrygin, École polytechnique

Abstract. We present a new algorithm to compute periodic (planar) straight-line drawings of toroidal graphs. Our algorithm is the first to achieve two important aesthetic criteria: the drawing fits in a straight rectangular frame, and the grid area is polynomial, precisely the grid size is $O(n^4 \times n^4)$. This solves the problem of the identifying of the vertices on the opposite sides - one of the main open problems in a recent paper by Duncan et al.

TBA

Manish Mandad, INRIA Titane

Abstract. TBA

The compressed annotation matrix: an efficient data structure for computing persistent cohomology

Clément Maria, INRIA Geometrica

Abstract. Persistent homology with coefficients in a field F coincides with the same for cohomology because of duality. We propose an implementation of a recently introduced algorithm for persistent cohomology that attaches annotation vectors with the simplices. We separate the representation of the simplicial complex from the representation of the cohomology groups, and introduce a new data structure for maintaining the annotation matrix, which is more compact and reduces substantially the amount of matrix operations. In addition, we propose a heuristic to simplify further the representation of the cohomology groups and improve both time and space complexities. We provide a theoretical analysis, as well as a detailed experimental study of our implementation and comparison with state-of-the-art software for persistent homology and cohomology. (Joint work with Jean-Daniel Boissonnat and Tamal K. Dey)

Multi-field persistent homology

Clément Maria, INRIA Geometrica

Abstract. We introduce the multi-field persistence diagram for the persistence homology of a filtered complex. It encodes compactly the superimposition of the persistence diagrams

of the complex with several field coefficients, and provides a substantially more precise description of the topology of the filtered complex. Specifically, the multi-field persistence diagram encodes the Betti numbers of integral homology and the prime divisors of the torsion coefficients of the underlying shape. Moreover, it enjoys similar stability properties as the ones of standard persistence diagrams, with the appropriate notion of distance. These properties make the multi-field persistence diagram a useful tool in computational topology. We develop algorithms to compute the multi-field persistence diagram of a filtered complex as well as distances between multi-field persistence diagrams. On the one hand, we introduce modular reconstruction for Gaussian eliminations, an algorithmic method of independent interest for matrix algorithms. We apply it to the persistent cohomology algorithm and obtain an output-sensitive algorithm in the size m' of the multi-field persistence diagram, where the size m' is the number of distinct points in the superimposition of diagrams with the several field coefficients. For a filtered complex of size m, the standard persistence cohomology algorithm computes the persistence diagram in any field in time $O(f(m))$; the modular reconstruction algorithm computes the multi-field persistence diagram of size m' in time $O(f(m')A)$, where A is a factor measuring the complexity of arithmetic operations. In practice, m' is close to m, and in particular $m' \ll r \times m$ when multi-field persistent homology is computed in r distinct fields. On the other hand, we generalize the computation of graph matching to the multi-field persistence setting in order to compute distances between multi-field persistence diagrams. For standard persistence diagrams of size $O(m)$, the distance between diagrams can be computed in time $O(g(m))$; our algorithm computes the distance between multi-field persistence diagrams of size $O(m')$ in time $O(g(m')A\sqrt{t})$, where t counts the number of distinct prime divisors of the torsion coefficients of the filtered complex. We provide theoretical and experimental analyses of the algorithms. In particular, we show that t and A are very small in real-world examples and the multi-field algorithms are, in practice, as fast as algorithms that compute persistent homology in a single field. (Joint work with Jean-Daniel Boissonnat)

On the reconstruction of convex sets from random normal measurements

Quentin Mérigot, CNRS - Université de Grenoble

Abstract. We study the problem of reconstructing a convex body using only a finite number of measurements of outer normal vectors. More precisely, we suppose that the normal vectors are measured at independent random locations uniformly distributed along the boundary of our convex set. Given a desired Hausdorff error η, we provide an upper bounds on the number of probes that one has to perform in order to obtain an η-approximation of this convex set with high probability. Our result rely on the stability theory related to Minkowski's theorem. Joint work with Hiba Abdallah.

Discrete systolic inequalities and decompositions of triangulated surfaces

Arnaud de Mesmay, École Normale Supérieure Paris

Abstract. How much cutting is needed to simplify the topology of a surface? We provide bounds for several instances of this question, for the minimum length of topologically non-trivial closed curves, pants decompositions, and cut graphs with a given combinatorial map in triangulated combinatorial surfaces (or their dual cross-metric counterpart). Our work builds upon Riemannian systolic inequalities, which bound the minimum length of non-trivial closed curves in terms of the genus and the area of the surface. We first describe a systematic way to translate Riemannian systolic inequalities to a discrete setting, and vice-versa. This implies a conjecture by Przytycka and Przytycki from 1993, a number of new systolic inequalities in the discrete setting, and the fact that a theorem of Hutchinson on the edge-width of triangulated surfaces and Gromov's systolic inequality for surfaces are essentially equivalent. Then we focus on topological decompositions of surfaces. Relying on ideas of Buser, we prove the existence of pants decompositions of length $O(g^{3/2}n^{1/2})$ for any triangulated combinatorial surface of genus g with n triangles, and describe an $O(gn)$-time algorithm to compute such a decomposition. Finally, we consider the problem of embedding a cut graph with a given combinatorial map on a given surface. Using random triangulations, we prove (essentially) that, for any choice of combinatorial map of cut graph, there are some surfaces on which any embedding has length superlinear in the number of triangles of the triangulated combinatorial surface.

Convergence rates for persistence diagram estimation in Topological Data Analysis

Bertrand Michel, UPMC

Abstract. Computational topology has recently seen an important development toward data analysis, giving birth to Topological Data Analysis. Persistent homology appears as a fundamental tool in this field. We show that the use of persistent homology can be naturally considered in general statistical frameworks . We establish convergence rates of persistence diagrams associated to data randomly sampled from any compact metric space to a well defined limit diagram encoding the topological features of the support of the measure from which the data have been sampled. Our approach relies on a recent and deep stability result for persistence that allows to relate our problem to support estimation problems (with respect to the Gromov-Hausdorff distance). Joint work with F. Chazal, M. Glisse and C. Labruère.

Numerical optimization of Dirichlet-Laplace eigenvalues on domains in surfaces

Régis Straubhaar, INRIA Geometrica

Abstract. In this talk, we are interested in optimizing the (first) eigenvalues of the Laplace-Beltrami operator. More precisely, the spectrum of the laplacian defined over a domain on a surface M (with Dirichlet boundary condition) is a positive sequence growing to infinity. A classical question is to find which domain minimizes the k-th eigenvalue among all domain of a given area and to find its value. In this talk, we go through the classical case $M = \mathbb{R}^2$, in

which theoretical results exist as well as numerical experimentations for higher eigenvalues. Then, we investigate numerically the case where M is the sphere, the hyperbolic disc and a hyperboloid.

Mirror-symmetry in images and 3D shapes

Viorica Patraucean, INRIA Geometrica

Abstract. Several psychophysical studies stress the importance of object symmetries in human perception when dealing with tasks related to object detection and recognition. Paradoxically, state-of-the-art methods in image and 3D shape analysis not only they reveal very limited use of object symmetry information, but they often regard symmetries as an issue. This talk will tackle this paradox, by addressing aspects linked to symmetry detection in images, and 3D shape matching in presence of symmetries. Specifically, a method for reducing the number of false positives in symmetry detection in images will be presented, as well as a 3D shape matching approach that solves the ambiguity induced by intrinsic symmetries in the shape matching problem.

Minkowski-type problems and optimal transport

Boris Thibert, Université de Grenoble

Abstract. In this article, we study the intersection (or union) of the convex hull of N confocal paraboloids (or ellipsoids) of revolution. This study is motivated by a Minkowski-type problem arising in geometric optics. We show that in each of the four cases, the combinatorics is given by the intersection of a power diagram with the unit sphere. We prove the complexity is $O(N)$ for the intersection of paraboloids and $\Omega(N^2)$ for the intersection and the union of ellipsoids. We provide an algorithm to compute these intersections using the exact geometric computation paradigm. This algorithm is optimal in the case of the intersection of ellipsoids and is used to solve numerically the far-field reflector problem. Joint work with P. Machado Manhães de Castro and Q. Mérigot.

Anisotropic Delaunay meshes of surfaces

Mael Rouxel-Labbé, INRIA Geometrica

Abstract. Anisotropic simplicial meshes are triangulations with elements elongated along prescribed directions. Anisotropic meshes have been shown to be well suited for interpolation of functions or solving PDEs. They can also significantly enhance the accuracy of a surface representation. Given a surface S endowed with a metric tensor field, we propose a new approach to generate an anisotropic mesh that approximates S with elements shaped according to the metric field. The algorithm relies on the well-established concepts of restricted Delaunay triangulation and Delaunay refinement and comes with theoretical guarantees. The star of each vertex in the output mesh is Delaunay for the metric attached to this vertex. Each facet has a good aspect ratio with respect to the metric specified at any of its vertices. The algorithm is easy to implement. It can mesh various types of surfaces like implicit surfaces, polyhedra or isosurfaces in 3D images. It can handle complicated geometries and topologies, and very anisotropic metric fields.

A convex body with chaotic hull

Rémy Thomasse, INRIA Geometrica

Abstract. The asymptotic behavior of the size of the convex hull of uniformly random points in a convex body in $\mathbb{R}^d$ is known for polytopes and smooth convex bodies. These are the lower and the upper bound for a general convex body K. In this talk, we exhibit an example of convex body whose size of the random convex hull alternates behavior close to the lower and to the upper bound for some ranges of random convex the number of points.